DAREK - Distance Aware Error for Kolmogorov Networks

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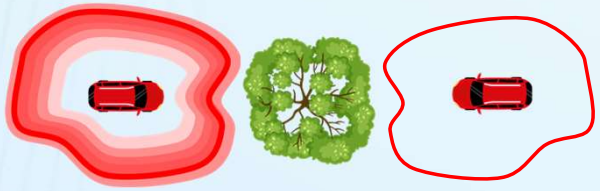
Introduction

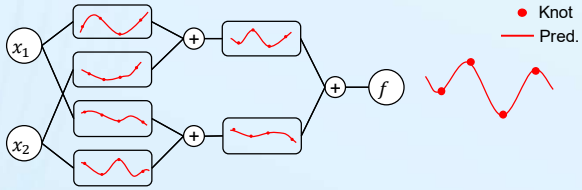
- Safety is important in autonomous decision-making systems
- Probabilistic uncertainty estimators are unbounded, require calibration, and poor scalability.
- DAREK is a distance aware, interpretable, and computationally efficient worst-case error bound

Probabilistic bound

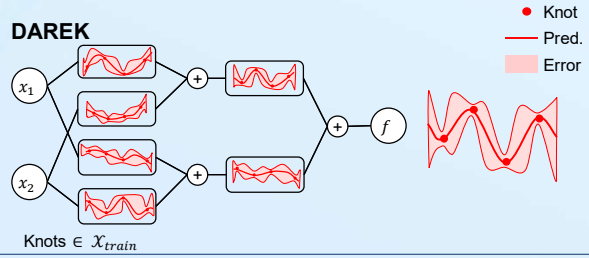
Worst-case bounded



Kolmogorov Arnold Networks (KAN)



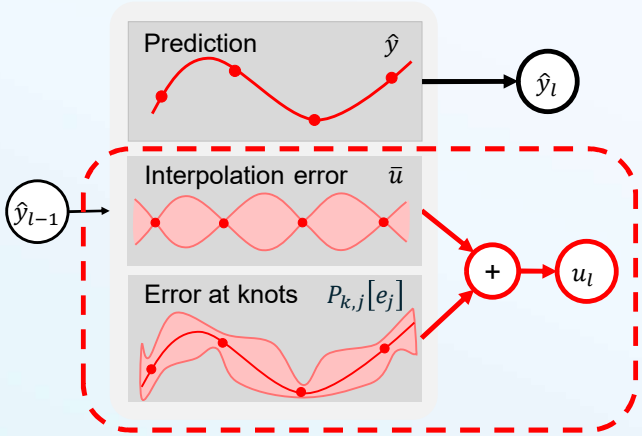
DAREK



Knots $\in x_{train}$

Method

Piecewise Polynomial error (PPE)

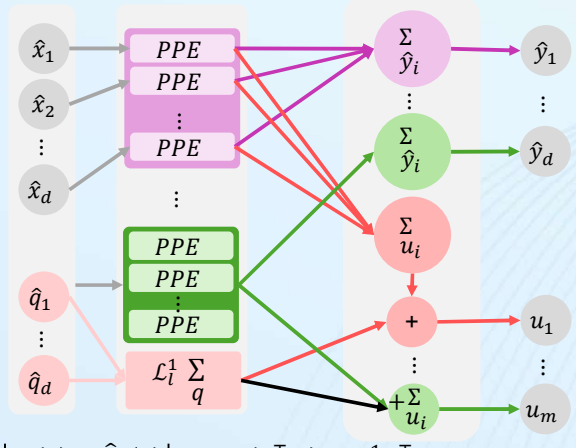


$$\bar{u}_f(x;\tau_{1:m}) \coloneqq \frac{\mathcal{L}_f^{k+1}}{(k+1)!}\left|\prod_{j=n}^{n+i-1}(x-\tau_j)\right| \quad \text{[Theorem 1]}$$

$$e_n^f(\tau_i) \coloneqq f(\tau_i) - \hat{f}_{[n]}(\tau_i)$$

$$u_f(x;\tau_{1:m}) \coloneqq \bar{u}_f(x) + \left|\mathcal{P}_{k,n}\left[e_n^f(\tau_{1:m})\right](x)\right| \quad \text{[Lemma 1]}$$

DAREK Layer (h_l)



$$\left|h_l(x) - \hat{h}_l(x)\right| \le u_{[l]}(\mathbf{1}_n^\top \hat{\boldsymbol{x}}) + \mathcal{L}_l^1 \mathbf{1}_d^\top \boldsymbol{q} \quad \text{[Theorem 2]}$$

Results

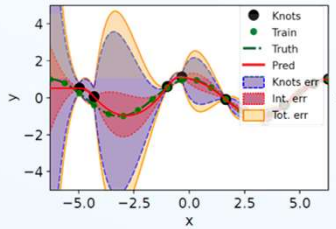


Fig. 1. The error bounds of a one-layer DAREK model on cosine function. The bound tightly close the error.

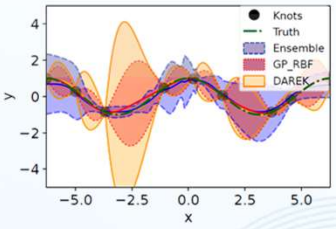


Fig. 2. The error bounds of a 2-layer DAREK model, Ensemble, and GP on cosine function. Ensemble and GP's uncertainty bounds are shown within the ±3σ range.

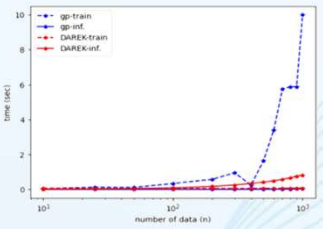


Fig. 3. The computation time of GP and DAREK for different number of sample points.

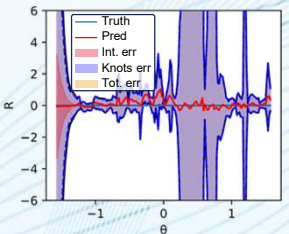


Fig. 4. The error bounds of a Sign Distance Function prediction. The estimation enclose the approximation error.

Conclusion

- **DAREK**, a novel framework for **error estimation** in spline based networks
- Provides **structured, interpretable, and computationally efficient worst-case error bounds**
- Uses **piecewise polynomial error estimation**, ensuring **tight, distance-aware error bounds**

Acknowledgements

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References

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- C. De Boor and C. De Boor, A practical guide to splines. springer New York, 1978, vol. 27.