Particle Flows for Source Localization in 3-D Using TDOA Measurements

Wenyu Zhang, Mohammad Javad Khojasteh, and Florian Meyer

wez078@ucsd.edu

Electrical and Computer Engineering Department
University of California San Diego

Contribution

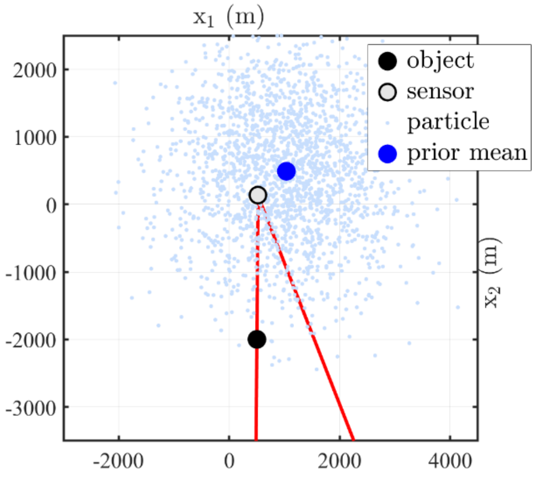
- We establish a Bayesian method for the localization of an unknown number of sources in 3-D using TDOA measurements that are subject to measurement-origin uncertainty
- We combine a Gaussian mixture representation with stochastic particle flow in a belief propagation framework to address challenges related to the nonlinear measurement model and the fact that posterior distributions can have hyperboloid shapes
- We validate our method in a challenging 3-D multisource localization problem with TDOA measurements and demonstrate robust and accurate localization performance

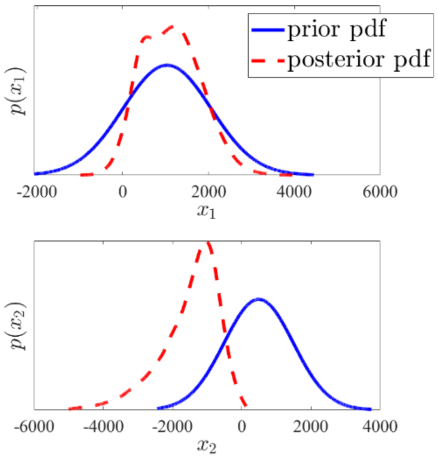
Particle Degeneracy

- Deterministic and Stochastic Particle Flow for 3-D TDOA measurements
- Belief Propagation with Particle Flow
- Simulation: Multi-Source Localization in 3-D

Particle Degeneracy

- Source localization in 2-D using TDOA measurements





$$\text{ESS} = \frac{1}{\sum_{i=1}^{N_s} w_i^2}, \text{ with } \sum_{i=1}^{N_s} w_i = 1$$

Method	Dimension	σ (secs.)	ESS
IS [1]	2D	1e-4	1.0003
AIS [2]	2D	1e-4	1997.2

Effective Sample Size (ESS) of Importance Sampling (IS) and Auxiliary Importance Sampling (AIS) using 2000 samples and a fixed measurement standard deviation σ

$$z = \frac{1}{c}\left(\|\boldsymbol{p} - \boldsymbol{q}^{(1)}\| - \|\boldsymbol{p} - \boldsymbol{q}^{(2)}\|\right) + v$$

source position: $\boldsymbol{p}$

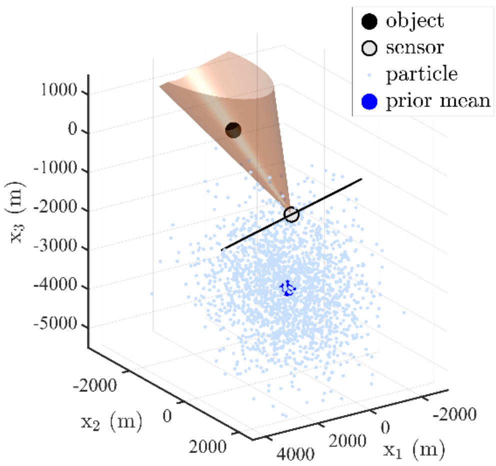
receiver positions: $\boldsymbol{q}^{(1)}, \boldsymbol{q}^{(2)}$

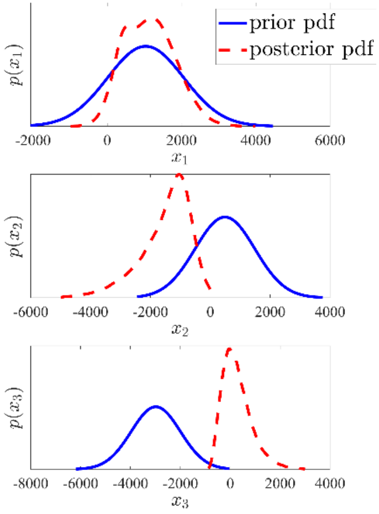
propagation speed: c

noise: $v \sim \mathcal{N}(0; \sigma)$

Particle Degeneracy

- Source localization in 3-D using TDOA measurements



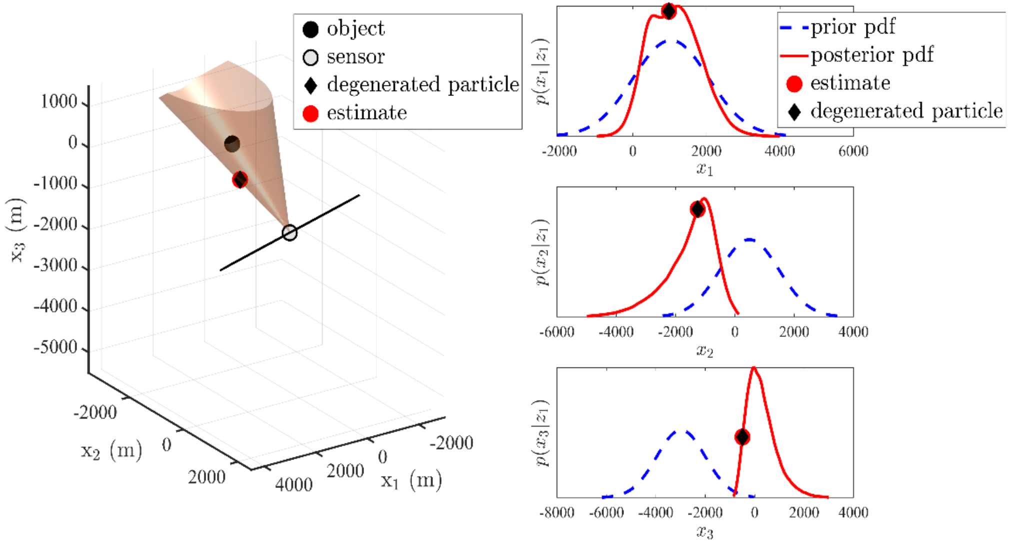


Method	Dimension	σ (secs.)	ESS
IS	2D	1e-4	1.0003
AIS	2D	1e-4	1997.2
AIS	3D	1e-4	5.2029
AIS	3D	1e-5	1.0000

Effective Sample Size (ESS) of Importance Sampling (IS) and Auxiliary Importance Sampling (AIS) using 2000 samples and different measurement standard deviations σ

Particle Degeneracy

- Source localization in 3-D using TDOA measurements



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Effective Sample Size (ESS) of Importance Sampling (IS) and Auxiliary Importance Sampling (AIS) using 2000 samples and different measurement standard deviations σ

M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Trans. Signal Process.*, vol. 50, no. 2, pp. 174–188, Feb. 2002.

M. K. Pitt and N. Shephard, "Filtering via simulation: Auxiliary particle filters," J. Am. Statist. Assoc., vol. 94, no. 446, pp. 590–599, Jun. 1999

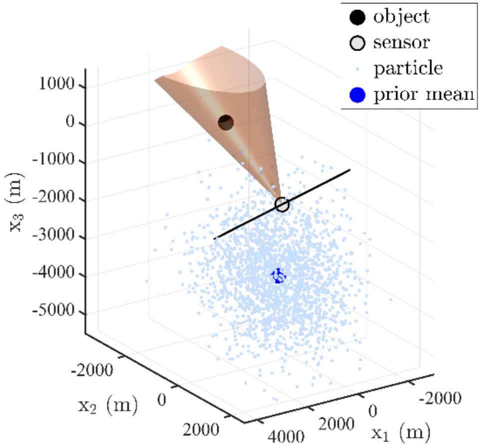
Particle Flow

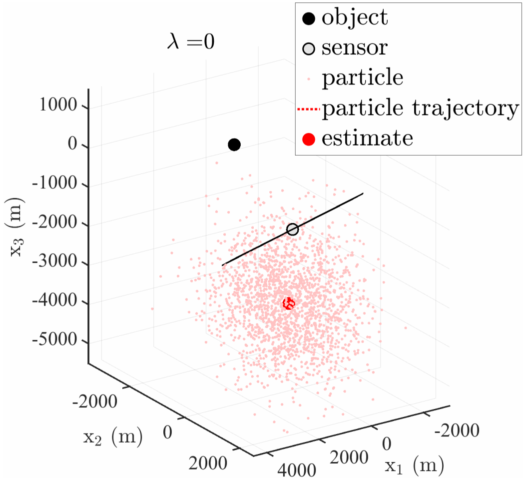
- Counter particle degeneracy using **particle flow (PF)**

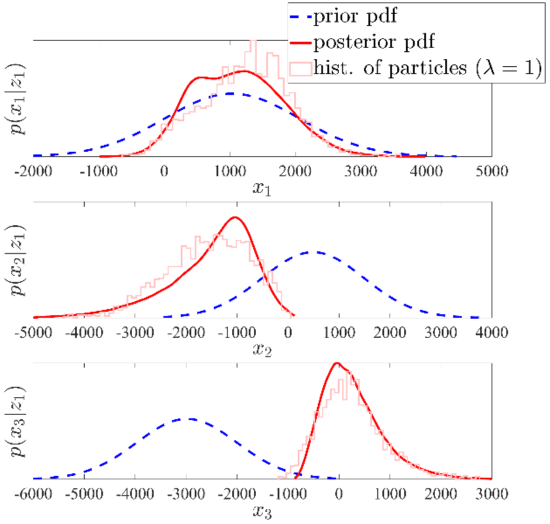
$f(\boldsymbol{x})$	prior pdf
$h(\boldsymbol{x}; z)$	likelihood function
λ	pseudo-time

$$\phi(\boldsymbol{x}, \lambda) = \log f(\boldsymbol{x}) + \lambda \log h(\boldsymbol{x}; z)$$

$$\phi(\boldsymbol{x}, 0) = \log f(\boldsymbol{x}) \quad \xrightarrow{\lambda : 0 \to 1} \quad \phi(\boldsymbol{x}, 1) \propto \log \pi(\boldsymbol{x}), \pi(\boldsymbol{x}) = \frac{f(\boldsymbol{x}) h(\boldsymbol{x}; z)}{p(z)}$$







W. Zhang and F. Meyer, "Multisensor multiobject tracking with improved sampling efficiency," *IEEE Trans. Signal Process.*, vol. 72, pp. 2036–2053, 2024.

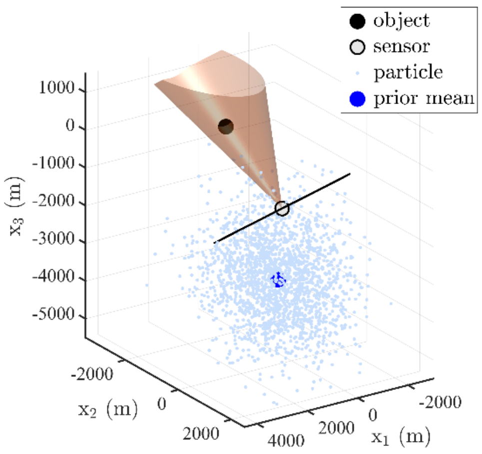
Particle Flow

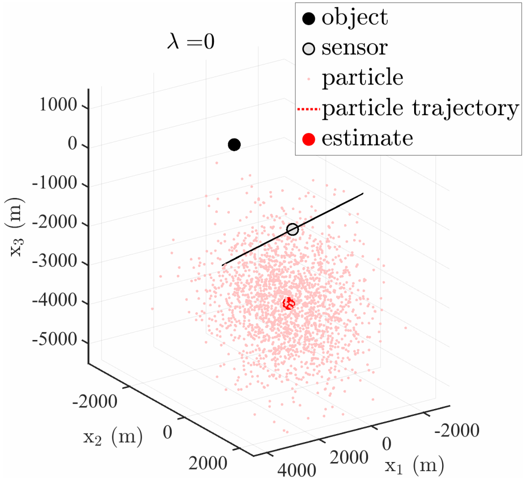
- Counter particle degeneracy using **particle flow (PF)**

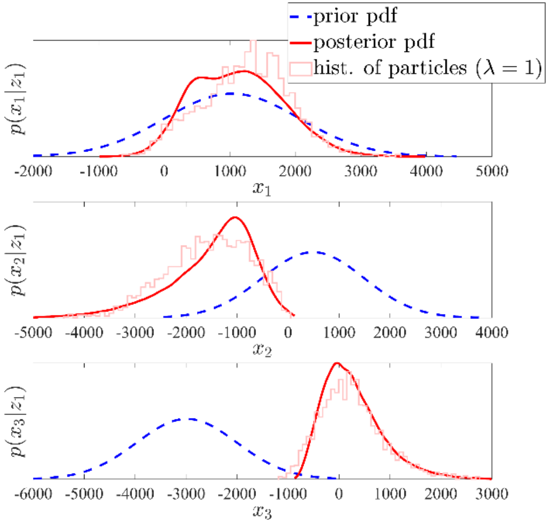
$$\phi(\boldsymbol{x}, \lambda) = \log f(\boldsymbol{x}) + \lambda \log h(\boldsymbol{x}; z)$$

$$\phi(\boldsymbol{x}, 0) = \log f(\boldsymbol{x}) \xrightarrow{\lambda : 0 \to 1}$$

Method	Dimension	σ (secs.)	ESS
PFL	3D	1e-5	1999.6
AIS	3D	1e-5	1.0000







W. Zhang and F. Meyer, "Multisensor multiobject tracking with improved sampling efficiency," *IEEE Trans. Signal Process.*, vol. 72, pp. 2036–2053, 2024.

Deterministic Flow

- Solve an ordinary differential equation (ODE)

$$\mathrm{d}\boldsymbol{x} = \zeta_{\mathrm{d}}(\boldsymbol{x}, \lambda)\mathrm{d}\lambda$$

following

$$\phi(\boldsymbol{x}, \lambda) = \log f(\boldsymbol{x}) + \lambda \log h(\boldsymbol{x}; z)$$

where $\zeta_{\mathrm{d}}(\boldsymbol{x}, \lambda) \in \mathbb{R}^N$ is the **drift**

- Exact Daum and Huang (EDH) flow
  - Gaussian prior $f(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}_0, \boldsymbol{P})$
  - Linear measurement model $\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$, where $H = \frac{\partial h(\boldsymbol{x}; z)}{\partial \boldsymbol{x}}$ and $\boldsymbol{v} \sim \mathcal{N}(\cdot; 0, \boldsymbol{R})$

$$\zeta_{\mathrm{d}}(\boldsymbol{x}, \lambda) = \boldsymbol{A}(\lambda)\boldsymbol{x} + \boldsymbol{b}(\lambda)$$

$$\boldsymbol{A}(\lambda) = -\frac{1}{2}\boldsymbol{P}\boldsymbol{H}^{\mathrm{T}}(\lambda \boldsymbol{H}\boldsymbol{P}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R})^{-1}\boldsymbol{H}$$

$$\boldsymbol{b}(\lambda) = (\boldsymbol{I} + 2\lambda \boldsymbol{A}(\lambda))\left[(\boldsymbol{I} + \lambda \boldsymbol{A}(\lambda))\boldsymbol{P}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{z} + \boldsymbol{A}(\lambda)\boldsymbol{\mu}_0\right]$$

- In the considered nonlinear TDOA localization problem we have to linearize the measurement model, i.e.,

F. Daum, J. Huang, and A. Noushin, "Exact particle flow for nonlinear filters," in Proc. SPIE-10, Apr. 2010, pp. 92–110.

Stochastic Flow

- Solve a stochastic differential equation (SDE)

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{\zeta}_{\mathrm{s}}(\boldsymbol{x}, \lambda)\mathrm{d}\lambda + \sqrt{\boldsymbol{Q}}\mathrm{d}\boldsymbol{w}_{\lambda}$$

following

$$\phi(\boldsymbol{x}, \lambda) = \log f(\boldsymbol{x}) + \lambda \log h(\boldsymbol{x}; z)$$

where $\boldsymbol{\zeta}_{\mathrm{s}}(\boldsymbol{x}, \lambda) \in \mathbb{R}^{N}$ is the **drift** and

$\boldsymbol{Q}(\boldsymbol{x}, \lambda) \in \mathbb{R}^{N \times N}$ the **diffusion**

- Gromov's flow

$$\boldsymbol{\zeta}_{\mathrm{s}}(\boldsymbol{x}, \lambda) = \boldsymbol{A}(\lambda)\boldsymbol{x} + \boldsymbol{b}(\lambda)$$

$$\boldsymbol{A}(\lambda) = -\left(\boldsymbol{P}^{-1} + \lambda \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}$$

$$\boldsymbol{b}(\lambda) = \left(\boldsymbol{P}^{-1} + \lambda \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{z}$$

$$\boldsymbol{Q}(\lambda) = (\boldsymbol{P}^{-1} + \lambda \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H})(\boldsymbol{P}^{-1} + \lambda \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}$$

F. Daum, J. Huang, and A. Noushin, "New theory and numerical results for Gromov's method for stochastic particle flow filters," in Proc. FUSION-18, 2018

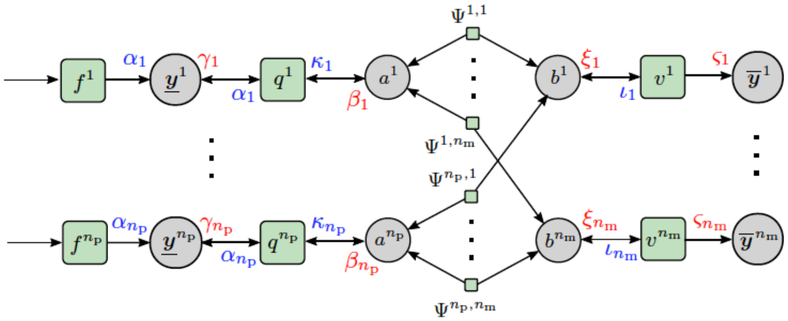
Outline

- Deterministic and Stochastic Particle Flow for 3-D TDOA measurements
- Belief Propagation with Particle Flow
- Simulation: Multi-Source Localization in 3-D

Challenges of 3D Multi-Source Localization

- 3D multi-source location from TDOA measurements is challenged by *measurement-origin uncertainty* and the fact that the *number of sources is unknown*

- To address these challenges, we adopt a framework of factor graphs and belief propagation (BP) originally developed for multiobject tracking
  - there is only a single time step
  - there are multiple receivers
  - every pair of receivers is considered a a "sensor" that provides TDOA measurements subject to MOU
  - sensors are processed sequentially



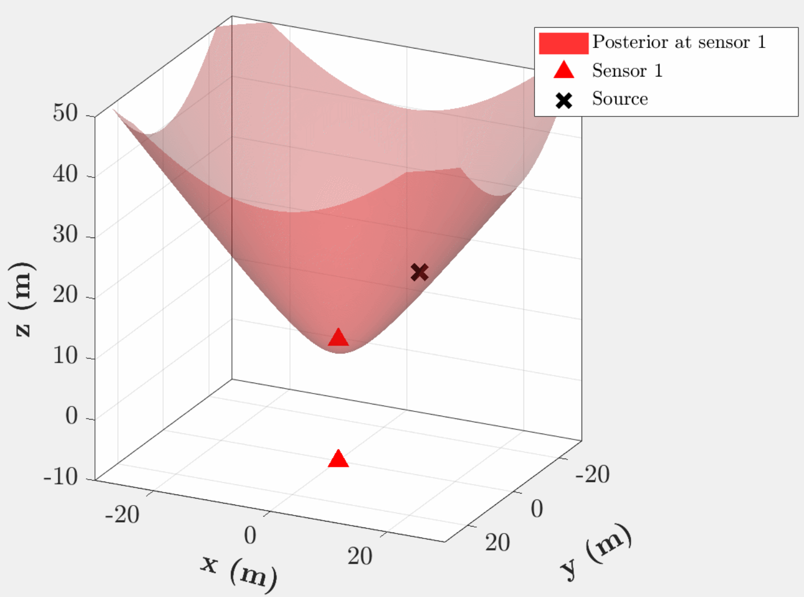
In the considered 3D problem, BP operations can suffer from particle degeneracy

F. Meyer, T. Kropfreiter, J. L. Williams, R. A. Lau, F. Hlawatsch, P. Braca, and M. Z. Win, "Message passing algorithms for scalable multitarget tracking," *Proc. IEEE*, 2018

Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion:A Handbook of Algorithms*. Storrs, CT: Yaakov Bar-Shalom, 2011.

BP Message Representation and Computation using PF

- Hyperboloid shaped distributions



- Gaussian mixture model (GMM)

$$f(\boldsymbol{x}) = \frac{1}{N_\mathrm{k}} \sum_{k=1}^{N_\mathrm{k}} \mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}_0^{(k)}, \boldsymbol{\Sigma}_0^{(k)})$$

- Computation of exemplary BP message using PF (sensor and source indexes are omitted)

$$\beta(m) = \int g(\boldsymbol{x}; z_m) f(\boldsymbol{x}) \mathrm{d}x \qquad m \in \{1, \ldots, M\}$$

$g(\boldsymbol{x}; z_m)$: likelihood ratio of measurement with index m

M: total number of measurements at current sensor

$$\left\{\left(\boldsymbol{x}_0^{(i)}, w_0^{(i)}\right)\right\}_{i=1}^{N_\mathrm{s}} \sim f(\boldsymbol{x})$$

$$\tilde{\beta}(m) = \sum_{i=1}^{N_\mathrm{s}} g\left(\boldsymbol{x}_0^{(i)}; z_m\right) w_0^{(i)}$$

$$\boldsymbol{x}_0^{(i)} \xrightarrow[\text{particle flow}]{\boldsymbol{z}_a} \boldsymbol{x}_1^{(i)}$$

$$\tilde{\beta}(m) = \sum_{i=1}^{N_\mathrm{s}} g\left(\boldsymbol{x}_1^{(i)}; z_m\right) w_1^{(i)} \qquad w_1^{(i)} = \frac{f(\boldsymbol{x}_1^{(i)})}{q(\boldsymbol{x}_1^{(i)})} w_0^{(i)}$$

Proposal Computation and Evaluation

- Compute Gaussian means and covariances iteratively using particle flow summarized for a single Gaussian components:

Given $f(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0 \triangleq \boldsymbol{P})$, for discrete time steps $0 = \lambda_0 < \cdots < \lambda_{N_\lambda} = 1$, we compute

$$\boldsymbol{\mu}_{\lambda_l} = \boldsymbol{\mu}_{\lambda_{l-1}} + \zeta_{\mathrm{s}}(\boldsymbol{\mu}_{\lambda_{l-1}}, \lambda_l)(\lambda_l - \lambda_{l-1})$$

$$\boldsymbol{\Sigma}_{\lambda_l} = [\boldsymbol{I} + (\lambda_l - \lambda_{l-1})\boldsymbol{A}(\lambda_l)]\boldsymbol{\Sigma}_{\lambda_{l-1}}[\boldsymbol{I} + (\lambda_l - \lambda_{l-1})\boldsymbol{A}(\lambda_l)]^{\mathrm{T}} + (\lambda_l - \lambda_{l-1})\boldsymbol{Q}(\lambda_l)$$

Finally, we obtain the proposal pdf $q(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$.

- Message approximation using GMM obtained from parallel particle flows

$$\tilde{\beta}(m) = \frac{1}{N_{\mathrm{k}}} \sum_{k=1}^{N_{\mathrm{k}}} \sum_{i=1}^{N_{\mathrm{p}}} g\big(\boldsymbol{x}_1^{(i,k)}; z_m\big) w_1^{(i,k)}$$

$$w_1^{(i,k)} = \frac{\mathcal{N}\big(\boldsymbol{x}_1^{(i,k)}; \boldsymbol{\mu}_0^{(k)}, \boldsymbol{\Sigma}_0^{(k)}\big)}{\mathcal{N}\big(\boldsymbol{x}_1^{(i,k)}; \boldsymbol{\mu}_1^{(k)}, \boldsymbol{\Sigma}_1^{(k)}\big)} w_0^{(i,k)}$$

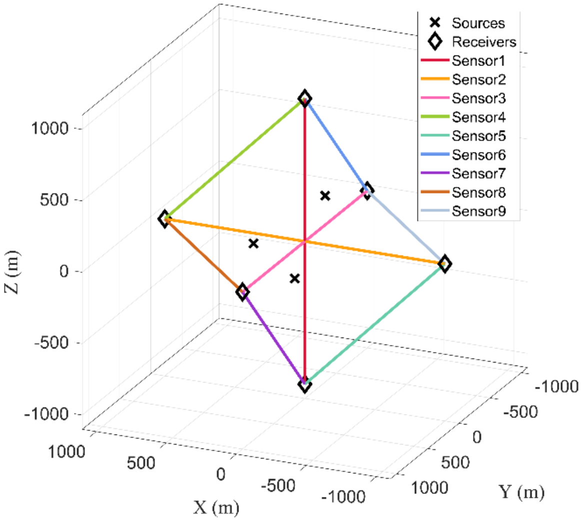
$f(\boldsymbol{x})$ prior

$q_k\big(\boldsymbol{x}_1^{(i,k)}\big)$ proposal resulting from flow k

Outline

- Deterministic and Stochastic Particle Flow for 3-D TDOA measurements
- Belief Propagation with Particle Flow
- Numerical Results

Simulation Scenario



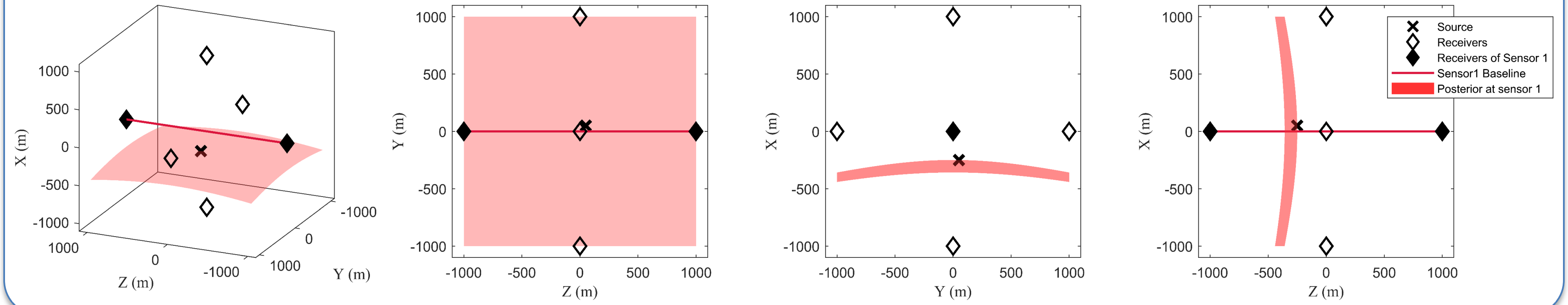
- TDOA measurement model and parameters

$$z = \frac{1}{c}\Big(\|\boldsymbol{p} - \boldsymbol{q}^{(1)}\| - \|\boldsymbol{p} - \boldsymbol{q}^{(2)}\|\Big) + v$$

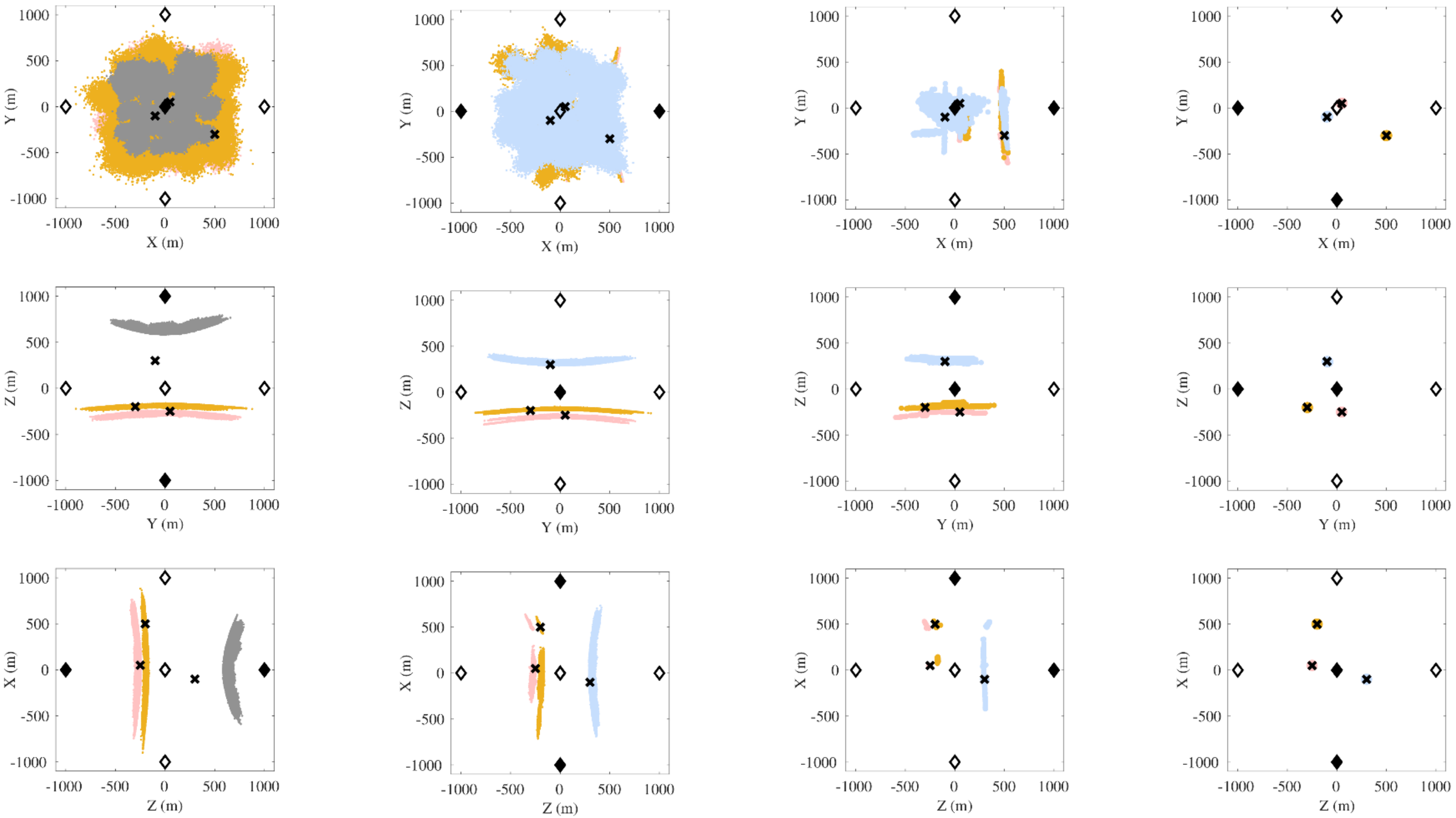
$\sigma_z = 1\text{e-}3$ s, $\quad c = 1500$ m/s

Mean number of false alarms: $\mu_{\mathrm{FA}} = 1$

- Distribution after update step of sensor 1 assuming a single TDOA measurement



Simulation Results

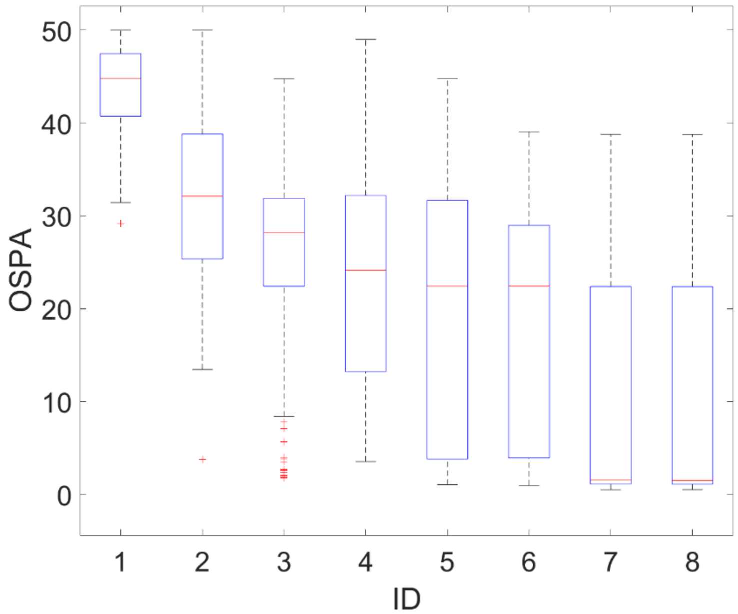


1st sensor | 2nd sensor | 4th sensor | 9th sensor

Simulation Results

ID	Method	$(N_k, \underline{N}_p, \overline{N}_p)$	OSPA [4]	Runtime (s)
1	IS	$(-, 2e6, 2e6)$	43.90	75.4
2	IS	$(-, 1e7, 1e7)$	32.70	443.3
3	EDH	$(100, 500, 30)$	25.17	196.9
4	LEDH	$(100, 500, 30)$	23.23	4934.2
5	EDH	$(100, 3e3, 500)$	20.57	379.6
6	EDH	$(100, 1e4, 1e4)$	19.58	2586.8
7	Gromov	$(100, 500, 30)$	10.43	568.8
8	Gromov	$(100, 3e3, 500)$	8.75	1356.1

Simulated mean OSPA error and runtime per run for different algorithms and system parameters, IS relies on conventional "bootstrap" importance sampling



Statistics of OSPA error for different algorithms; each column corresponds to a different method; the method IDs are defined in the table on the right

D. Schuhmacher, B.-T. Vo, and B.-N. Vo, "A consistent metric for performance evaluation of multi-object filters," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3447–3457, Aug. 2008.

Conclusion

- We proposed a 3-D source localization method that relies on TDOA measurements
- Our approach combines a Gaussian mixture representation with stochastic particle flow in a belief propagation framework
- Our results show significant performance improvements compared to a reference method that relies on conventional “bootstrap” importance sampling, especially when stochastic particle flow is employed
- Future research includes the application to real-world problems, e.g., the localization of marine mammals underwater