Event-triggered Stabilization over Digital Channels

Mohammad Javad Khojasteh

Wireless Information and Network Sciences Laboratory
Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
mkhojast@mit.edu

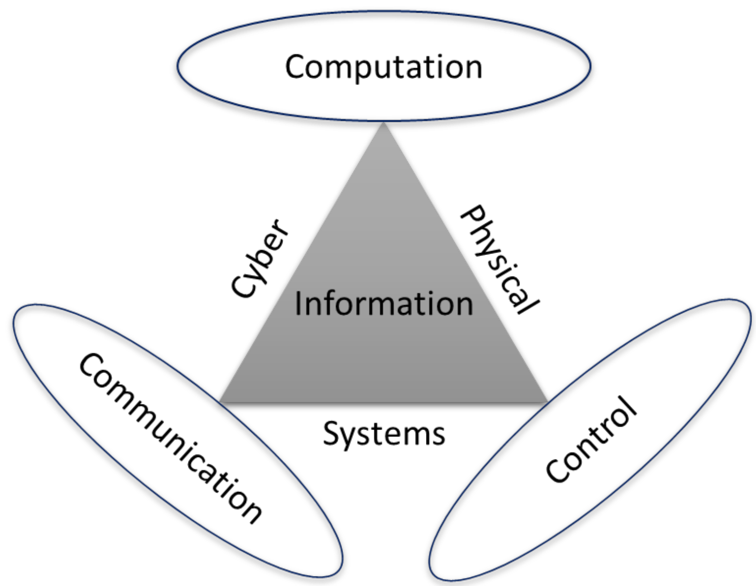
Outline

- Preliminaries
  - cyber-physical systems
  - data-rate theorem
- Event-triggered stabilization over digital channels
  - scalar systems
  - experimental Validation
  - Zeno Behavior
  - event-triggered vs. time-triggered
  - vector systems
  - exponential convergence
- Discussion and future work

PRELIMINARIES

Cyber-Physical Systems (CPS)

- Largely regarded as the next-generation engineering systems
- Integration of computing, communication, and control
- Arising in diverse areas such as robotics, energy, and transportation



Cloud Robots and Automation Systems

- An example of CPS
  - an emerging field in robotics and automation
  - cloud enables robots to use shared resources
  - feedback loop is closed over a communication channel
    - noisy and subject to delay

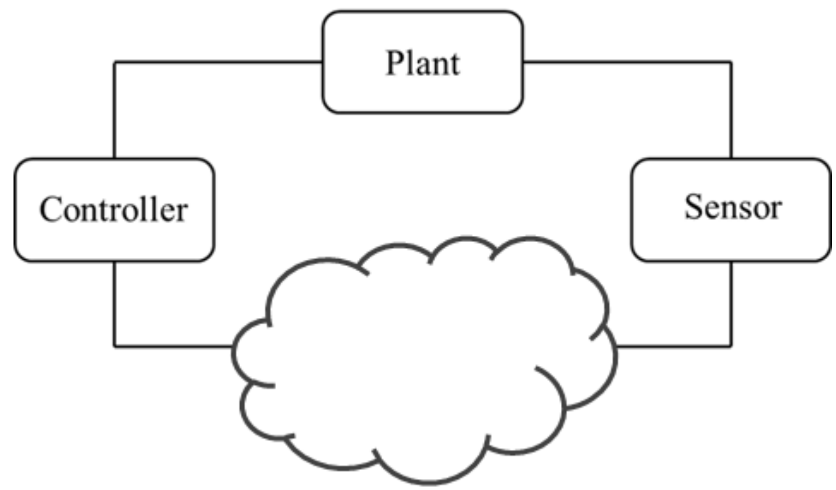
Networked Control Systems

- Plant is scalar

$$\dot{X} = aX(t) + bU(t) + W(t)$$

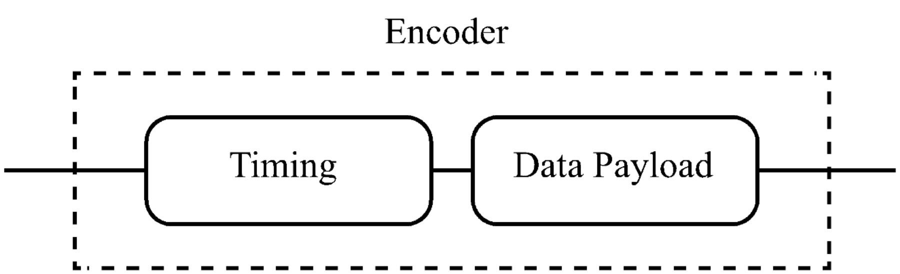
$$|W(t)| \leq m$$

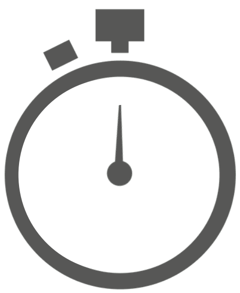
- Plant is unstable
- Communication channel is subjected to a finite data rate and bounded unknown delay



Communication Channel

Timing Information





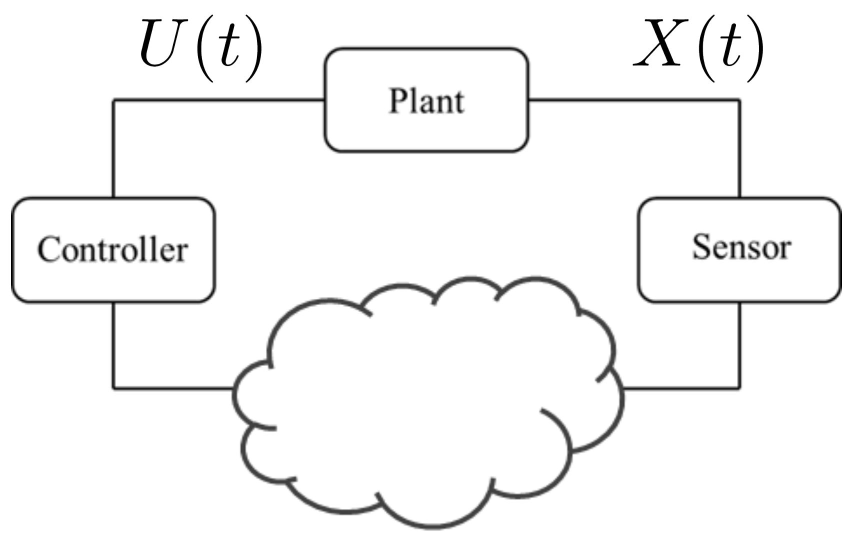
Timing Information
+
Data Payload

Delay

2.0 s

1.0 s

0.5 s



Communication Channel

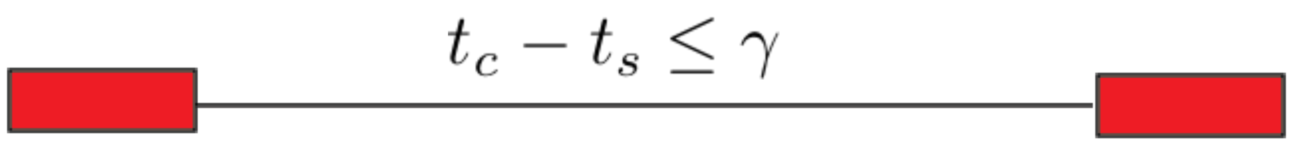
1 0 1 1

1 0 0 1

0 0 1 1

Transmission with Delay

- Packet transmission time t_s
- Packet reception time t_c
- Delay $t_c - t_s \leq \gamma$

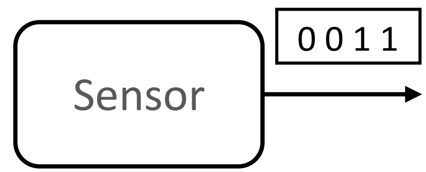


Information Rate

- $b_s(t)$ number of bits in data payload transmitted up to time t
  - information transmission rate

$$R_s = \limsup_{t \to \infty} \frac{b_s(t)}{t}$$

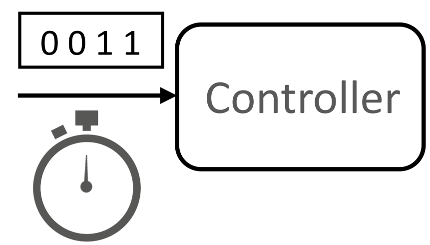
  - the rate at which the sensor transmits data payload



- $b_c(t)$ be the amount of information measured in bits included in data payload and timing information received at the controller until time t
  - information access rate

$$R_c = \limsup_{t \to \infty} \frac{b_c(t)}{t}$$

  - the rate at which controller receives information

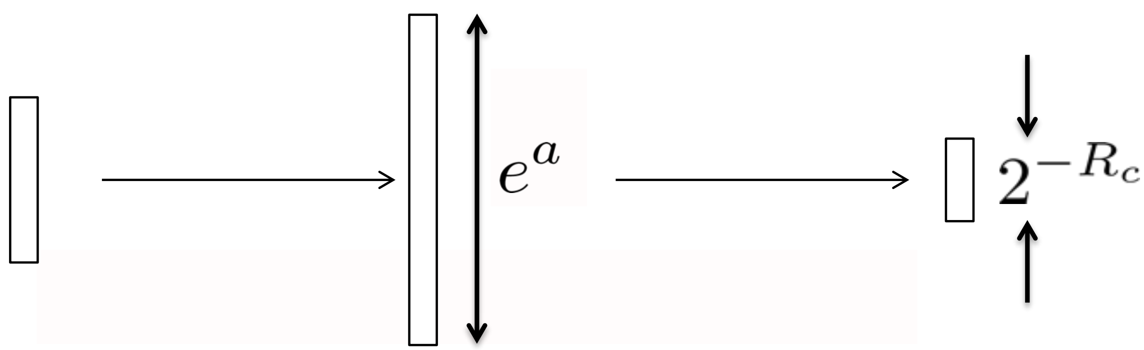


Data-rate Theorem

- We can stabilize the system if and only if the information access rate

$$R_c > \frac{a}{\ln 2}$$ → entropy rate of the plant

state uncertainty



Data-rate Theorem

- Balance between production and consumption of information

- This information can be supplied to the controller by data payload as well as timing

$$R_c > \frac{a}{\ln 2} \qquad\qquad R_s?$$

EVENT-TRIGGERED STABILIZATION OVER DIGITAL CHANNELS

Event-triggering Review

- Periodic control is the most common and perhaps simplest solution for digital systems.

  – Step 1: Good Dog
  – Step 2: Good Dog
  – Step 3: Bad Dog
  – Step 4: Good Dog

  .

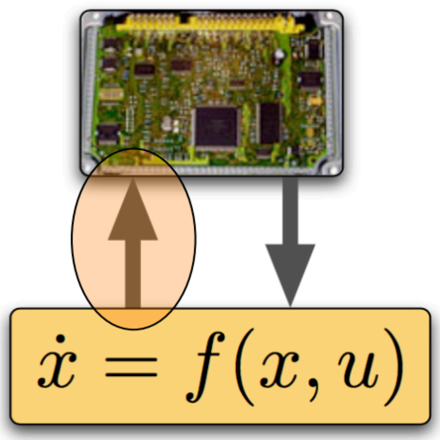
  .

  .

Genibo SD Robot Dog

Event-triggering Review

- In CPS we need to use the shared resources efficiently
  - periodic control can be inefficient
  - event-triggered control transmit sensory data in an opportunistic manner



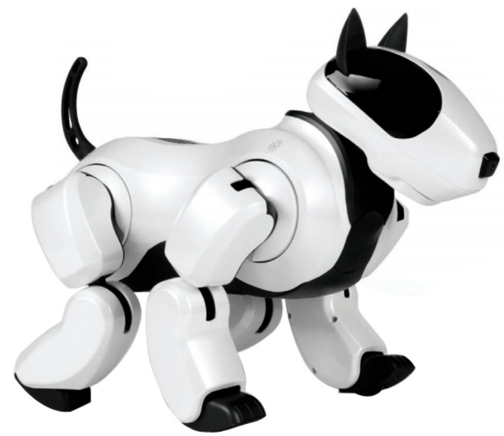
Event-triggering Review

- The main concept of event-triggered control is to transmit sensory data only when needed
  - Step 1: --
  - Step 2: --
  - Step 3: Bad Dog
  - Step 4: --

  .

  .

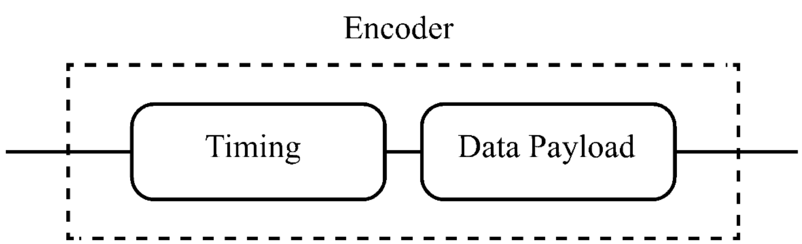
  .



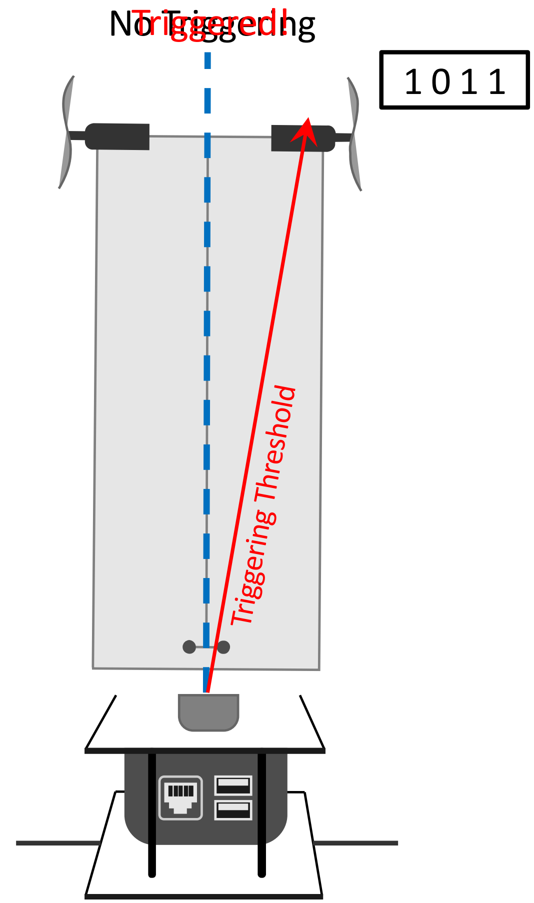
- "Wise men speak because they have something to say" — Plato

State Dependent Timing Information Encoding

- Our goal is to propose an event-triggering strategy that utilizes timing information by transmitting in a state-dependent fashion.



  - intuitive example
    - stabilization of an inverted pendulum over a digital communication channel



Input-to-state Practical Stability (ISPS)

- Encoding-decoding scheme, which encodes information in timing via event-triggering, to achieve ISpS

$$|X(t)| \leq \beta(|X(0)|, t) + \psi(|W|_t) + \chi(\gamma) + \zeta(|W|_t, \gamma).$$

$$\beta \in \mathcal{KL} \qquad \psi \in \mathcal{K}_\infty(0) \qquad \chi \in \mathcal{K}_\infty(d) \qquad \zeta \in \mathcal{K}^2_\infty(0, d')$$

$$|W|_t = \sup_{s \in [0,t]} |W(s)|$$

  - for a fixed γ, this definition reduces to the standard notion of ISpS (Z-P Jiang, A. R. Teel, L. Praly- 94 and Sharon, Liberzon- 12)

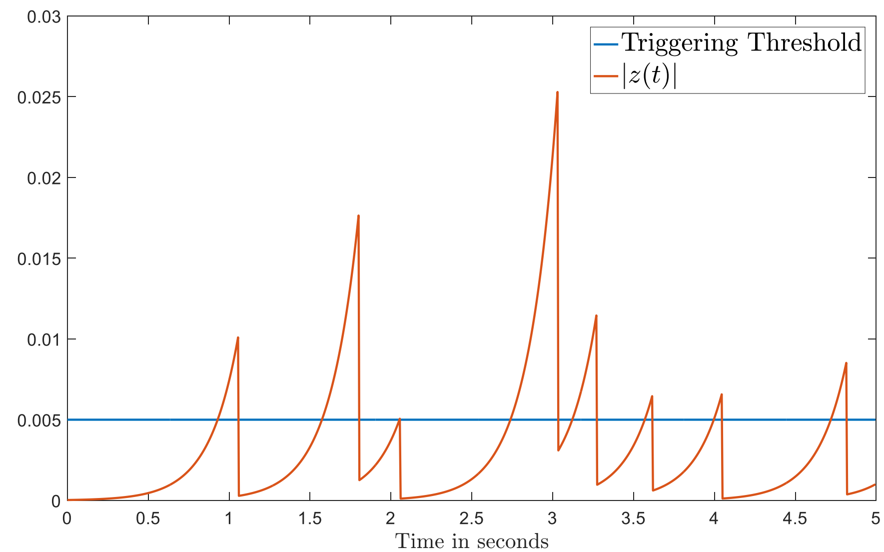
  - given that the initial condition, delay, and system disturbances are bounded, ISpS implies that the state must be bounded at all times

State Estimation Error

- Plant $$\dot{X} = aX(t) + bU(t) + W(t)$$
- $\hat{X}(t)$ the state estimation constructed at the controller
  - inter-triggering times $$\dot{\hat{X}}(t) = A\hat{X}(t) + BU(t), \quad t \in (t_c^k, t_c^{k+1})$$
- We assume the sensor can also compute the same estimate $\hat{X}(t)$ via a feedback acknowledgment
  - communication via control input
  - control input is known at the sensor and it jumps only at each reception times
- State estimation error $$Z(t) = X(t) - \hat{X}(t)$$

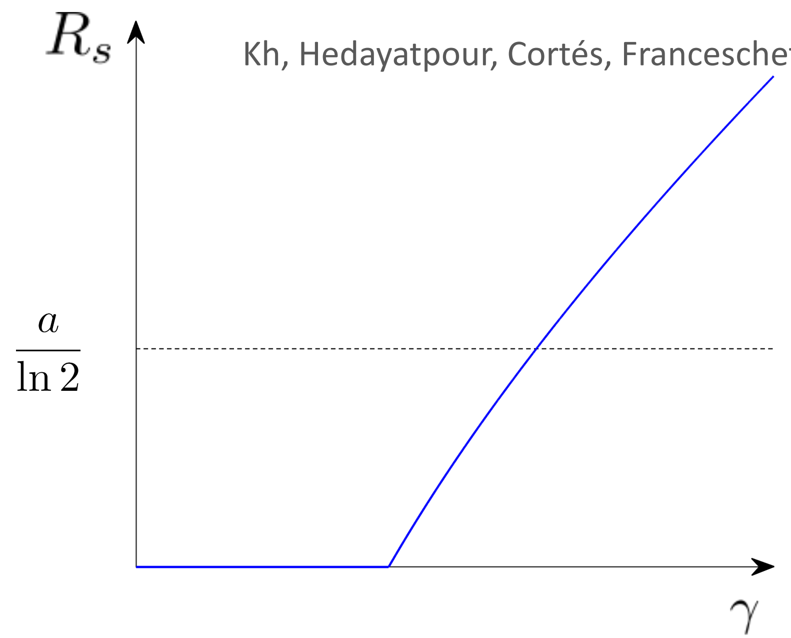
Triggering Strategy

- Triggering criterion $|Z(t_s)| = J$
  - triggering threshold J
  - $|Z(t_c^+)|$ is always below the triggering threshold
  - $|Z(t)|$ is bounded



Information Transmission Rate

- Required information transmission rate vs delay upper bound
  - small values of delay
    - timing information is substantial
    - R_s is arbitrarily close to zero
  - as delay increases
    - timing information becomes out of date
    - R_s begin to increase
  - large value of the delay
    - uncertainty at the controller increases
    - state estimation error should be below the threshold at the reception time
    - R_s exceeds the rate imposed by the data-rate theorem



Kh, Hedayatpour, Cortés, Franceschetti- 21

Challenges

- Packet size
  - necessary Condition

$$\#\text{bits} \geq \log \frac{m(\text{uncertainty set})}{m(\text{covering ball})}$$

  - sufficient condition
    - we designed an encoding-decoding scheme
      - encode a quantized version of the triggering time in the data payload and timing

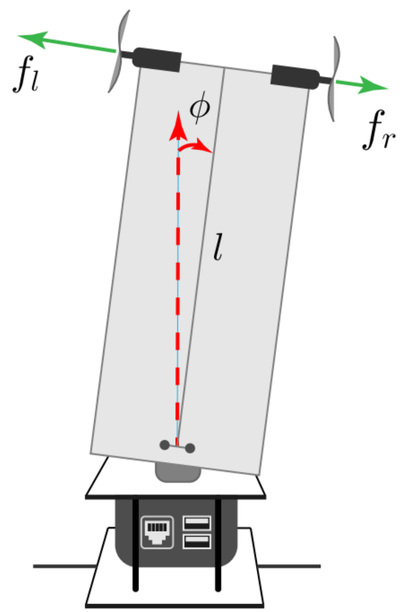
- Triggering rate

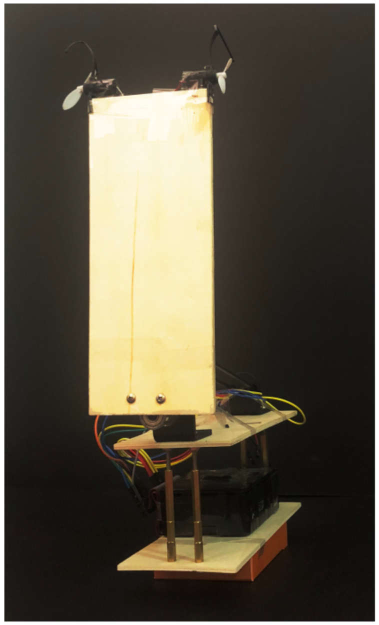
$$\text{Frequency} = \limsup_{N\to\infty} \frac{N}{\sum_{k=1}^{N} k^{th}\text{inter-event time}}$$

  - necessary Condition: lower bound
  - sufficient condition: upper bound

Experimental Validation

- Laboratory-scale inverted pendulum
  - using linearized model
  - stabilization around unstable equilibrium point

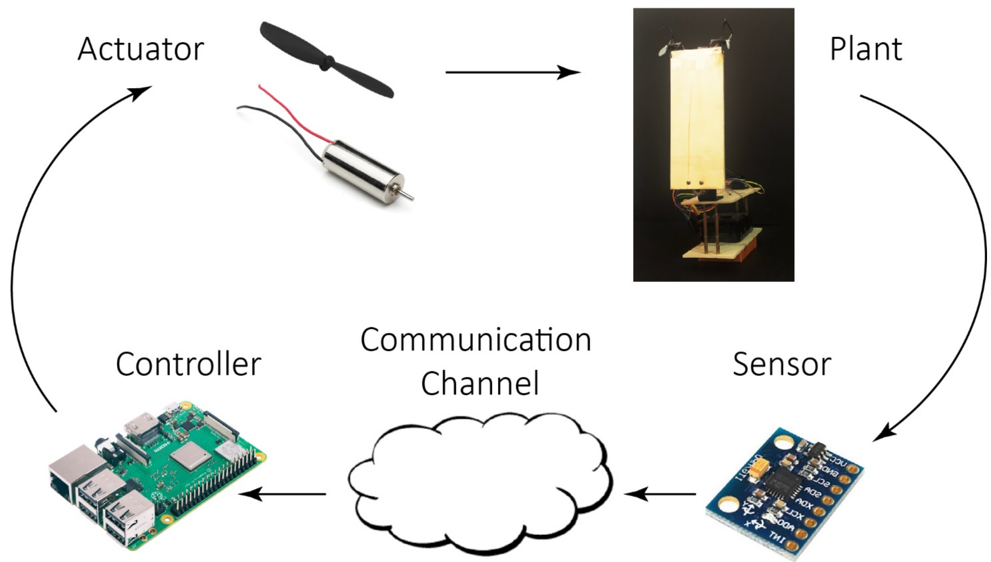




Experimental Validation

- Off-the-shelf components
  - raspberry Pi model 3
  - two small DC motors
  - two identical propellers
  - MEMS sensor
    - 3-axis accelerometer
    - 3-axis gyroscope
  - complimentary filter

  - details of these experiments
    - Kh, Hedayatpour, Franceschetti- 19



Experiment 1

- Delay upper bound
  - 2 sampling times
- Packet size
  - 1 bit
- Number of samples
  - 6541
- Number of triggering
  - 170

- Information transmission rate < Entropy rate of the system

8.6633 bit/sec < 10.5461 bit/sec

Experiment 2

- Delay upper bound
  - 3 sampling times
- Packet size
  - 3 bit
- Number of samples
  - 6333
- Number of triggering
  - 146

- Information transmission rate > Entropy rate of the system

23.0526 bit/sec > 10.5461 bit/sec

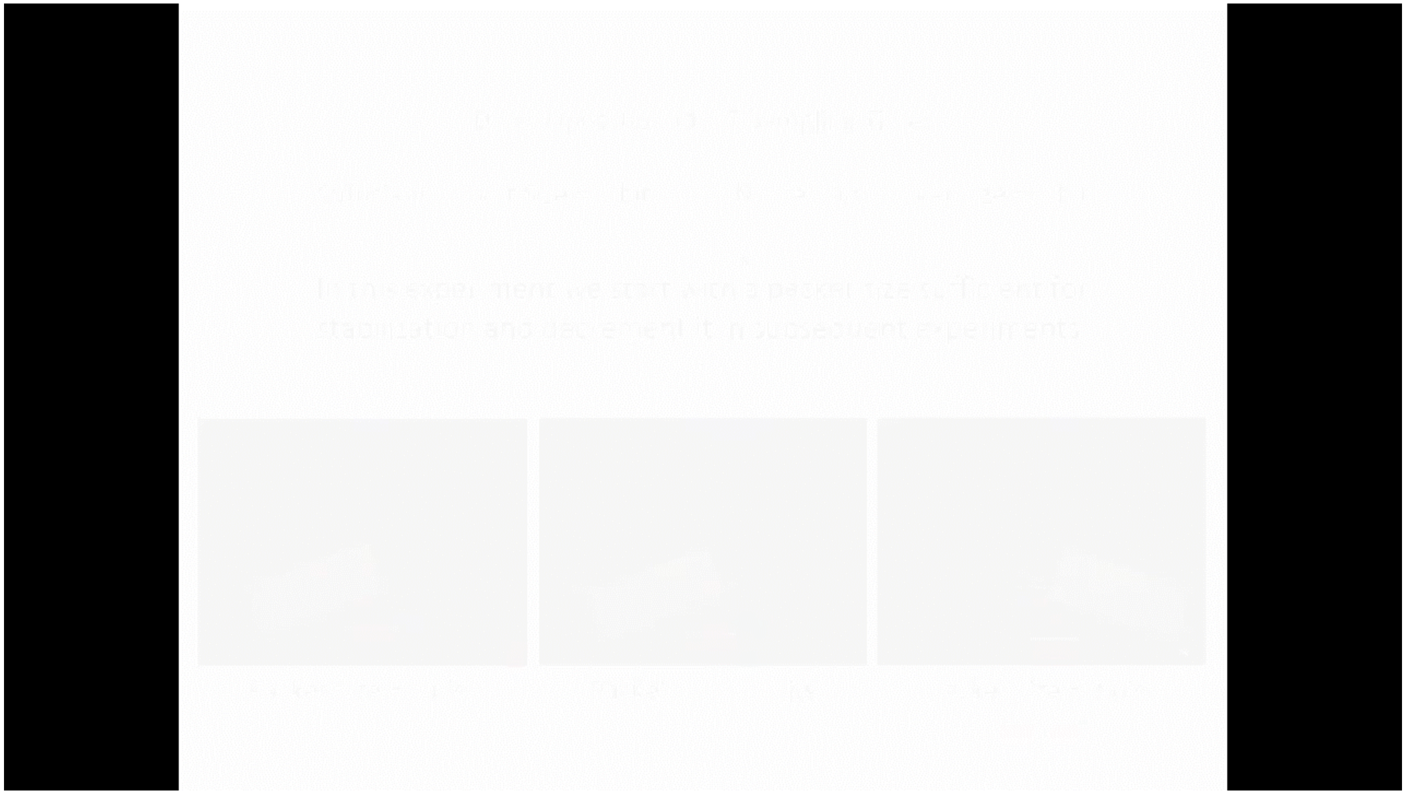
Experiment 3

- Delay upper bound
  - 7 sampling times

- Packet Size
  - sufficient packet size:
    - 5 bit
  - necessary packet size:
    - 1 bit

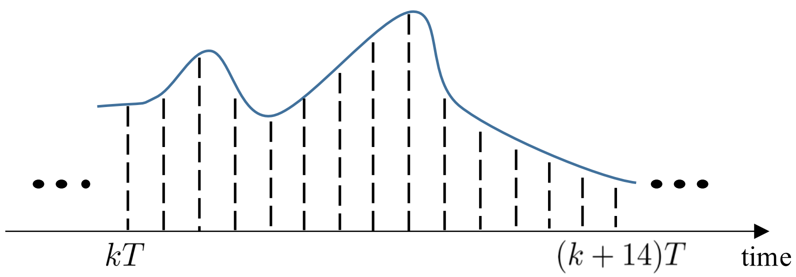
- In this experiment we start with a packet size sufficient for stabilization and decrease it in subsequent experiments

Experiment 3

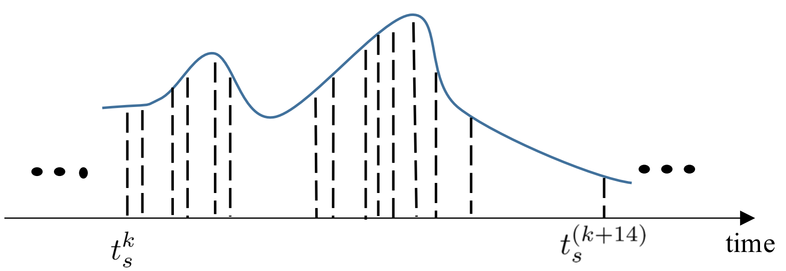


Zeno Behavior

- Periodic control
  - Equal-distance sampling

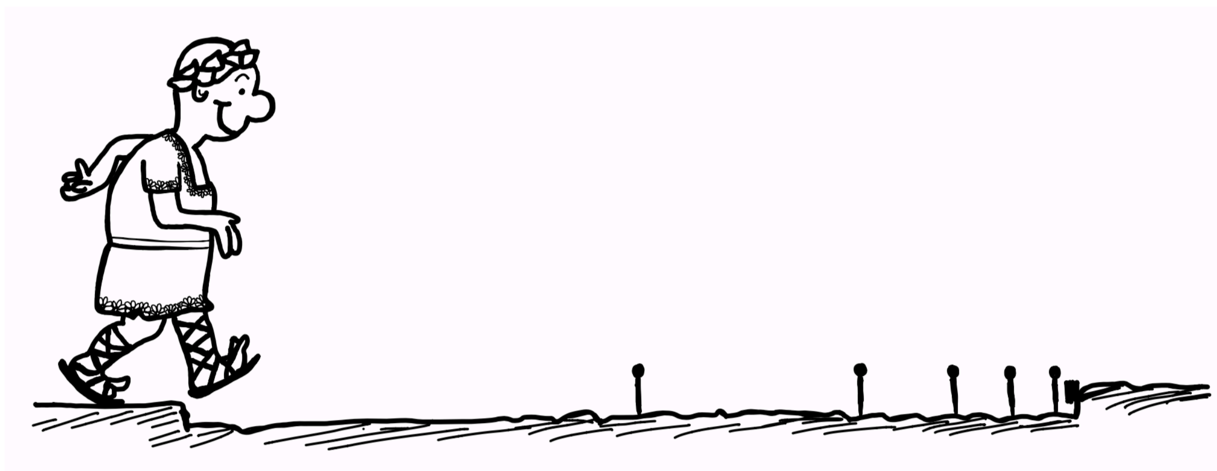


- Event-triggered control
  - sporadic sampling
  - hybrid phenomenon
    - Zeno behavior



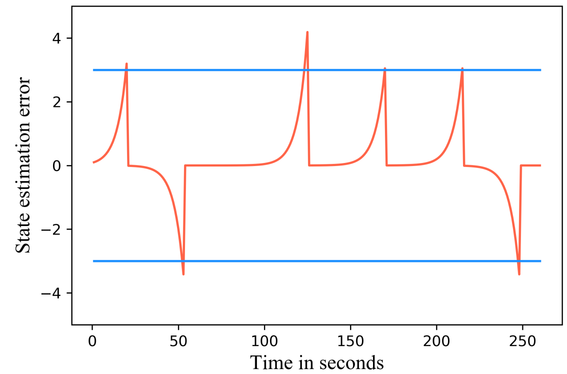
Zeno Behavior

- A paradox by ancient Greek philosopher Zeno of Elea
  - "That which is in locomotion must arrive at the half-way stage before it arrives at the goal."
  - We should never be able to reach any destination!

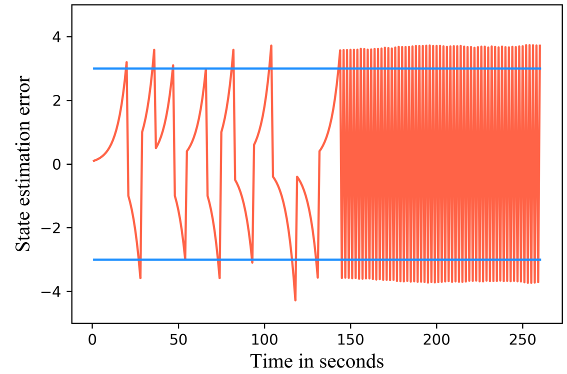


Zeno Behavior

- Normal realization



- Zeno realization
  - degenerate behavior of some event-triggering strategies
  - infinite number of triggering events occurring in a finite amount of time

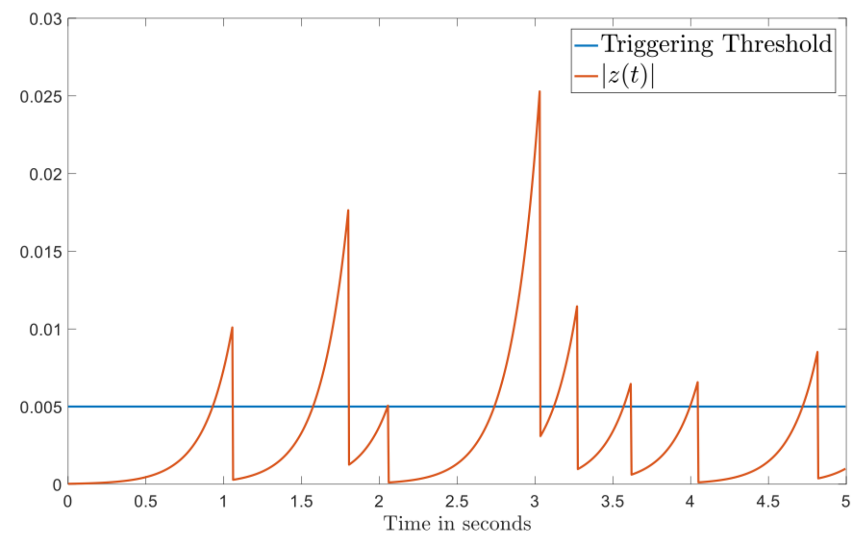


Zeno Behavior

- Event-triggering strategies
  - guarantee stability
  - rule out the Zeno behavior
- Design packet size
  - for $0 < \rho_0 < 1$

$$|z(t_c^+)| \leq \rho_0 J$$

  - uniform lower bound on the inter-triggering times



Time-triggering vs Event-triggering

- We compared our results against information access rate $R_c > \dfrac{a}{\ln 2}$
- In a time-triggered strategy $R_s?$
  - time-triggered strategy

$$t_s^0 = 0, \quad t_s^{k+1} = t_s^k + (\lfloor \Delta_k / T \rfloor + 1)T$$

  - similar to our event-triggering setup a packet is transmitted only after the previous packet is received.

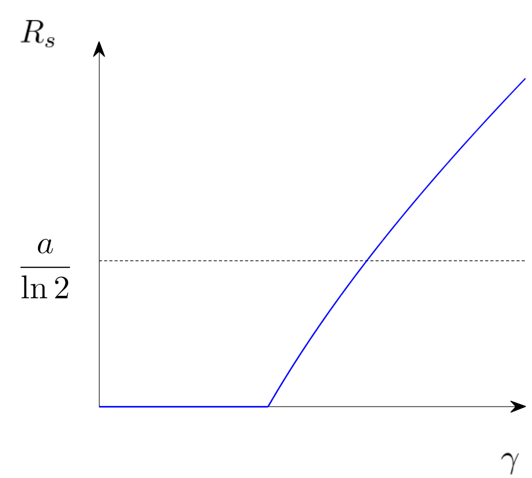
Time-triggering vs Event-triggering

- Time-triggering strategies
  - delay dependent
  - does not exploit timing information

$$R_s \geq \frac{a(\lfloor \frac{\gamma}{T} \rfloor + 1)}{\ln 2}$$

Kh, Tallapragada, Cortés, Franceschetti- 17

- Event-triggering strategies
  - state and delay dependent
    - transmit sensory data only when needed
    - exploit timing information



Vector Systems

- Data-rate theorem

$$R_c > \frac{Tr(A)}{\ln 2}$$

- Time-Triggering

$$R_s \geq \frac{Tr(A)(\lfloor \frac{\gamma}{T} \rfloor + 1)}{\ln 2}$$

- Event-Triggering

Vector Systems

- Triggering criterion
  - various ways $||z(t_s)||_2 = v(t_s)$

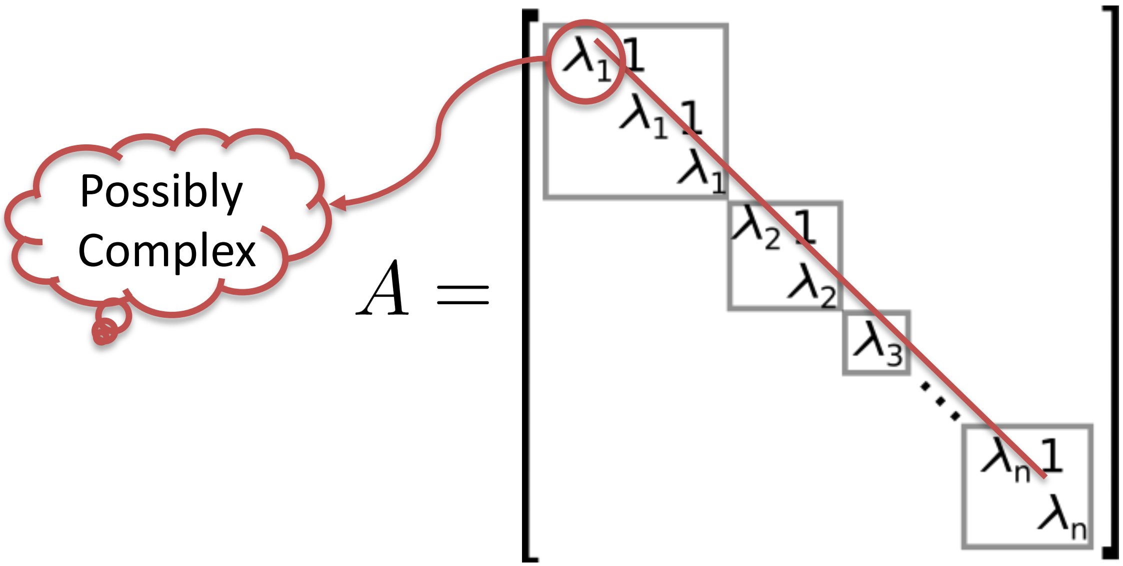
    - coordinate by coordinate analysis $|z_i(t)| = J_i$
      - this corresponds to treating the n-dimensional system as n scalar coupled systems.

Vector Systems: Communication Channel

- We assume that there are n parallel finite-rate digital communication channels between each coordinate of the system and the controller, each subject to unknown, bounded delay

- In the case of a single communication channel, we can consider the same triggering strategy, but an additional $\lceil \log n \rceil$ bits should be appended at the beginning of each packet to identify the coordinate it belongs to

Vector Systems: Jordan Block



– off-diagonal ones make coupling between states
  - Kh, Tallapragada, Cortés, Franceschetti- 20

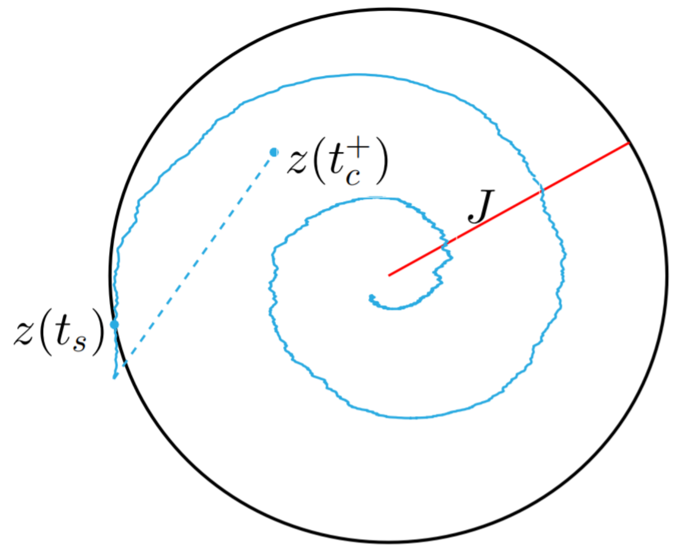
Extension to Complex Linear Systems

- Plant $$\dot{X} = aX(t) + bU(t) + W(t)$$
  - bounded disturbances $$\|W(t)\| \leq m$$
  - data-rate theorem extension $$R_c > \frac{2Re(a)}{\ln 2}$$
    - this information can be supplied to the controller by data payload as well as timing

$$R_s?$$

Triggering Strategy

- Triggering criterion $\|Z(t_s)\| = J$
  - triggering radius J
  - $\|Z(t_c^+)\|$ is always inside the triggering circle
  - $\|Z(t)\|$ is bounded



The Encoding

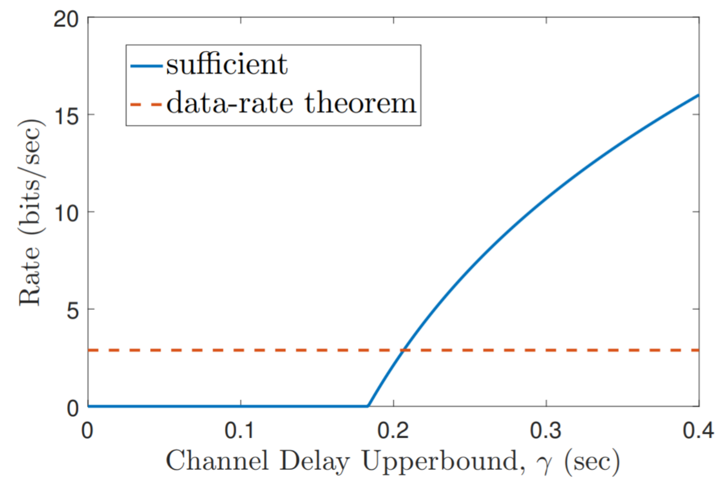
10001010101001101010010	10101010010110101010100101000

A uniform quantization of the phase at which the state estimation error hits the triggering circle

A quantized version of triggering time which is constructed like our encoding process for linear scalar systems.

Information Transmission Rate

- Required information transmission rate for stabilization
  - similar to scalar real plant
    - for small values of the delay, is smaller than the rate required by the data-rate theorem R_s



Kh, Hedayatpour, Cortés, Franceschetti- 21

Exponential Convergence

- Exponential convergence of the estimation error or the plant state
  - $\forall t > 0 \qquad |z(t)| \leq |z(0)|\, e^{-\sigma t}$ or $\forall t > 0 \qquad |x(t)| \leq |x(0)|\, e^{-\sigma t}$

$$R_c \geq \frac{A + \sigma}{\ln 2}$$

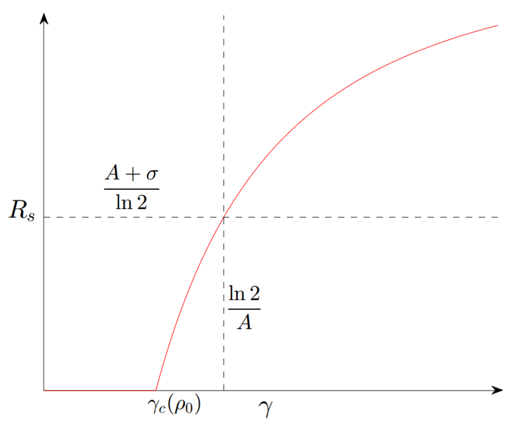
  - the access rate should be larger than entropy rate of the plant + convergence rate
    - Kh, Tallapragada, Cortés, Franceschetti- 17
    - estimation entropy (Liberzon, Mitra -17)

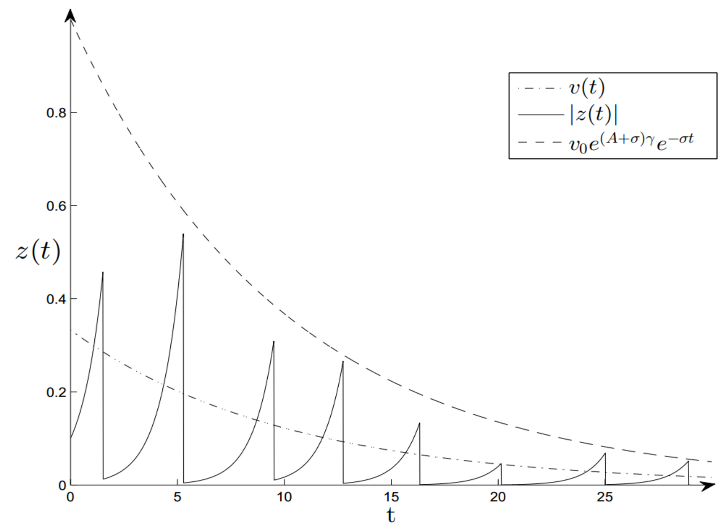
Exponential Convergence

- Time-triggering

$$R_s \geq \frac{(a+\sigma)(\lfloor \frac{\gamma}{T} \rfloor + 1)}{\ln 2}$$

- Event-triggering





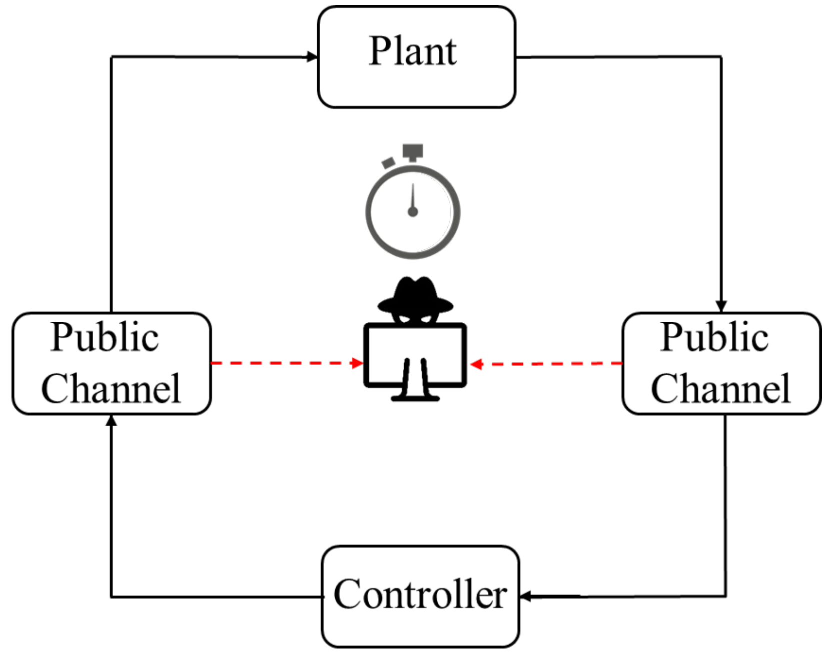
- Kh, Tallapragada, Cortés, Franceschetti- 20

DISCUSSION AND FUTURE WORK

Security and Privacy Issues

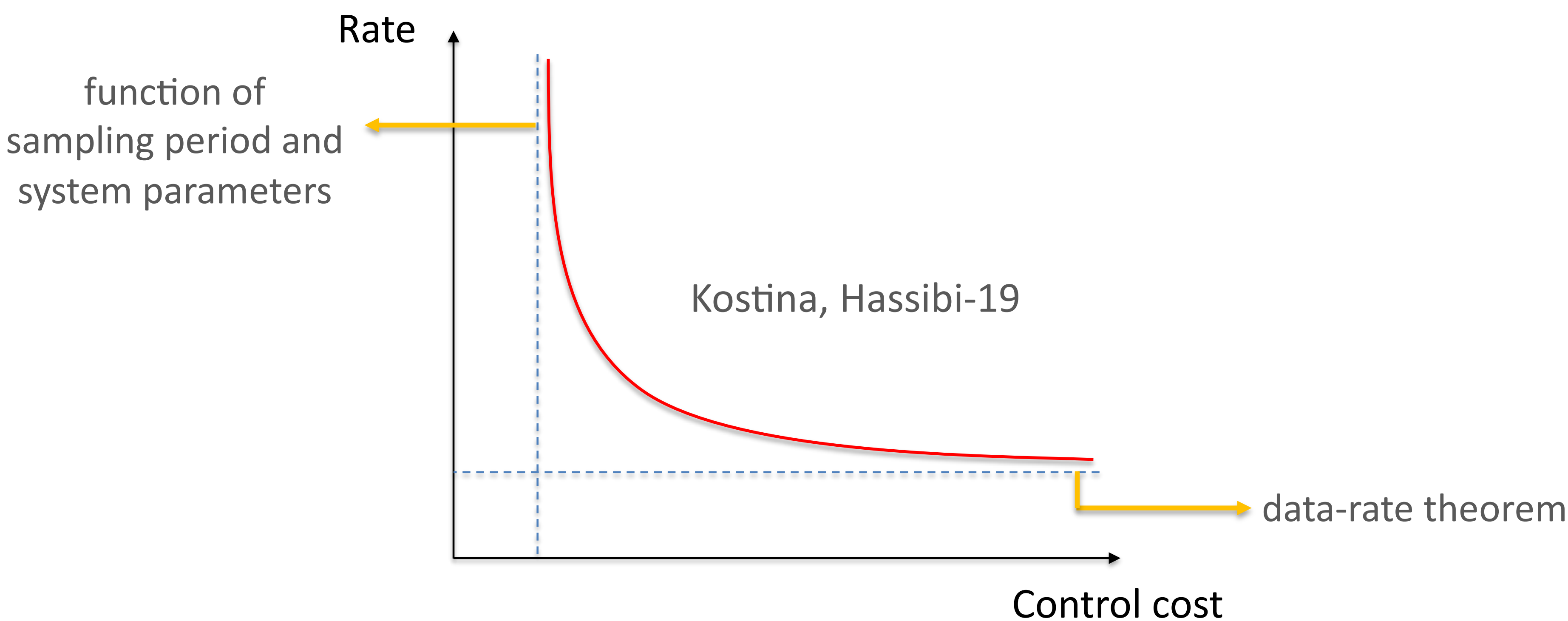
- Adversaries might take advantage of the inherent timing information in even triggering

- In context of
  - differential privacy
    - Cortes et al, CDC 2016
  - learning-based attacks
    - Khojasteh et al, TCNS 2021.



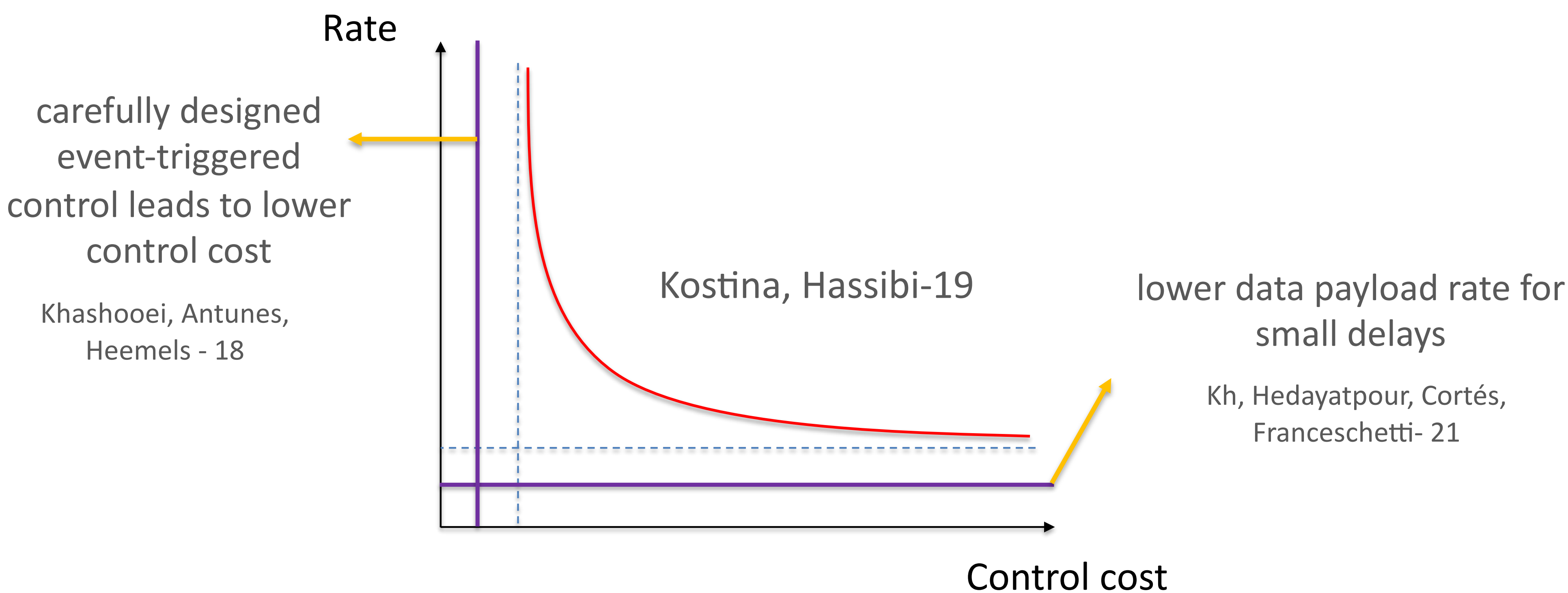
Rate-cost Tradeoffs in Periodic Control

- Appropriate communication rate to achieve a control objective



Rate-cost Tradeoffs in Event-based Control

- The event-triggering can improve this results in two aspects



Nonlinear Systems

- Plant

$$\dot{X} = f(X(t), U(t), W(t))$$

  - bounded disturbances

$$|W(t)| \leq m$$

  - locally Lipschitz

$$|f(X, U, W) - f(\hat{X}, U, 0)| \leq L_x|X - \hat{X}| + L_w|W|$$

Nonlinear Systems

- There exists a control policy which renders the dynamic ISS with respect to estimation error and system disturbances.

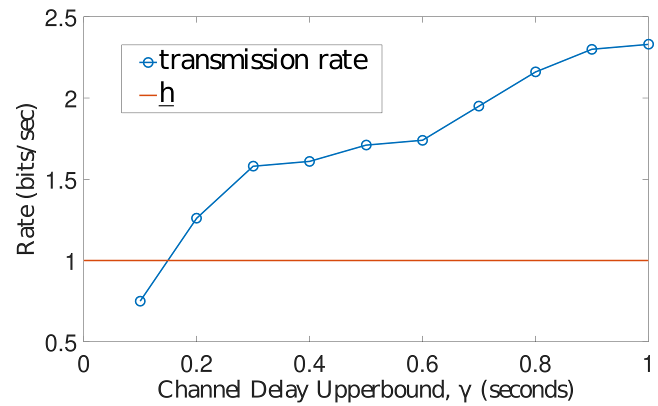
$$|X(t)| \leq \beta'(|X(0)|, t) + \Pi'(|Z|_t) + \psi'(|W|_t)$$

$$\beta' \in \mathcal{KL} \qquad \Pi' \in \mathcal{K}_\infty(0) \qquad \psi' \in \mathcal{K}_\infty(0)$$

$$Z(t) = X(t) - \hat{X}(t)$$

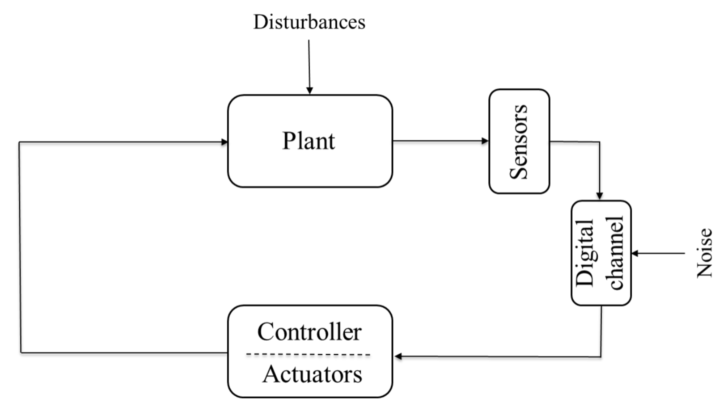
$$|W|_t = \sup_{s \in [0,t]} |W(s)|$$

- similar to linear plant
  - for small delay, we are below data-rate theorem
    - Kh, Hedayatpour, Franceschetti- 19
- extension to vector system
- relaxing the above assumption
  - similar to Hespanha, Liberzon, Teel - 08 for periodic control



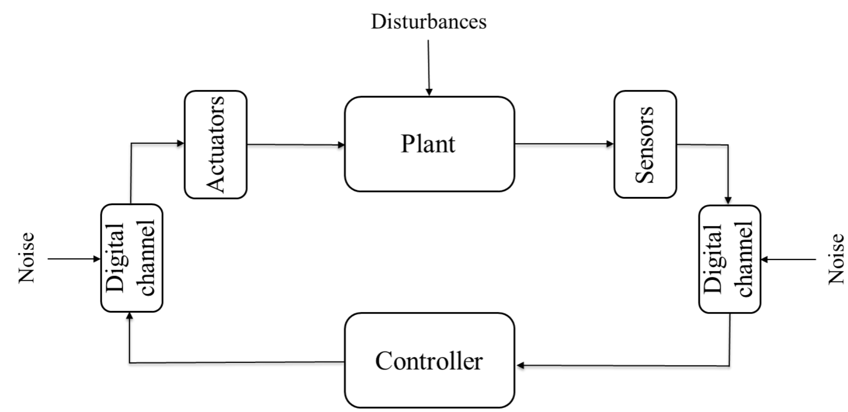
Uplink and Downlink Channels

- Data-rate theorems focused on Uplink
  - main bottleneck in mobile robots
    - week on-board transmitter
  - controller is co-located with the actuators
  - serve as causal feedback
    - acknowledge the received symbol to the sensor
    - plant is the communication medium
      - communication via control input



Uplink and Downlink Channels

- A digital channel in the downlink between the controller and the plant
  - extension of theses data-rate results



References

- Franceschetti M, Khojasteh M J, Win M Z
  - Information Flow in Event-Based Control of Cyber-Physical Systems
    - Book chapter (Computation-aware Algorithmic Design for Cyber-Physical Systems), In progress
- Khojasteh M J, Hedayatpour M, Cortes J, Franceschetti M
  - Exploiting timing information in event-triggered stabilization of linear systems with disturbances
    - IEEE Transactions on Control of Network Systems, 2021
- Khojasteh M J, Tallapragada P, Cortes J, Franceschetti M
  - The value of timing information in event-triggered control
    - IEEE Transactions on Automatic Control, 2020
- Khojasteh M J, Hedayatpour M, Franceschetti M
  - Theory and implementation of event-triggered stabilization over digital channels
    - IEEE 58th Annual Conference on Decision and Control, 2019
- Khojasteh M J, Tallapragada P, Cortes J, Franceschetti M
  - Time-triggering versus event-triggering control over communication channels
    - IEEE 56th Annual Conference on Decision and Control, 2017