Node Deployment under Position Uncertainty for Network Localization

Mohammad Javad Khojasteh, Augustin A. Saucan, Zhenyu Liu, Andrea Conti, and Moe Z. Win

Laboratory for Information and Decision Systems
Massachusetts Institute of Technology

2022 IEEE International Conference on Communications

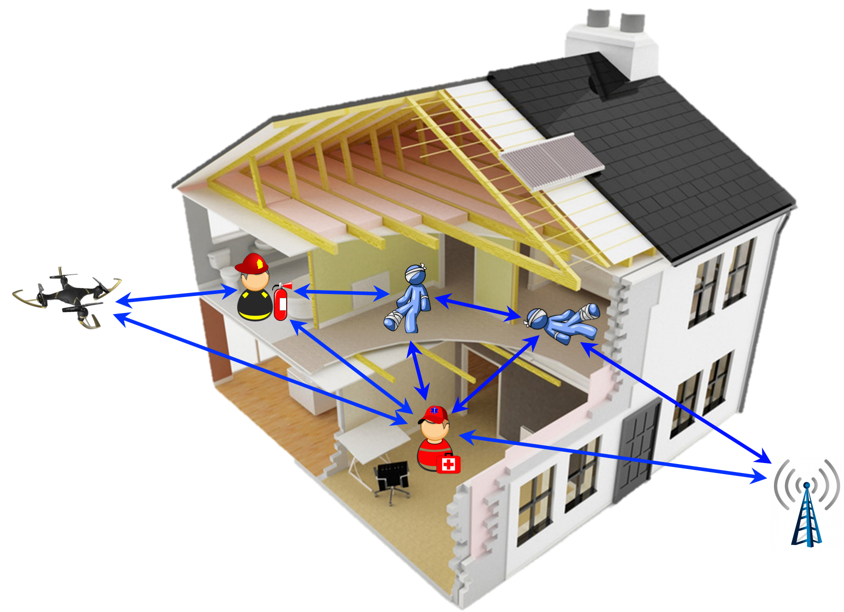
Outline

- Preliminaries
- Performance Metrics for Network Localization
- Relaxed Node Deployment
- Optimization-based Solution
- Final Remarks

PRELIMINARIES

Network Localization

- Location awareness is essential for many applications
  - crowdsensing, smart cities, and Internet-of-Things
- Network localization enables the collection of position information, where a network of sensing nodes are used to aid in localizing its members
  - situational awareness in first responder operations
- The localization performance strongly depends on the wireless environment and network's geometry



Examples

- Ocean-of-things (OoT)
  - floating devices aim to provide continuous maritime surveillance and ocean situational awareness
    - the sensing nodes (floats) are deployed off the coast of Italy

- Indoor positioning systems

A. Saucan and M. Z. Win, Information-seeking sensor selection for Ocean-of-Things, *IEEE Internet Things J*, vol. 7, no. 10, pp. 10072–10088, 2020.

B. Teague, Z. Liu, F. Meyer, A. Conti, and M. Z. Win, Network localization and navigation with scalable inference and efficient operation, *IEEE Trans. Mobile Comput.*, 2022, to appear.

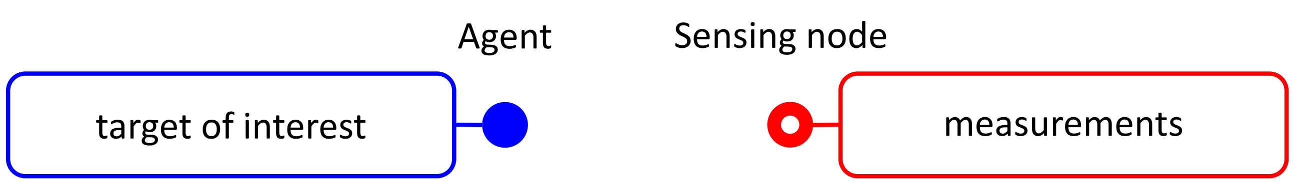
Network Localization

- Localization network comprised of N_b sensing nodes
  - with index set $\mathcal{N}_\mathrm{b} = \{1, 2, \ldots, N_\mathrm{b}\}$ at positions $\{\boldsymbol{q}_j\}_{j \in \mathcal{N}_\mathrm{b}}$
  - we assume N_b is even, and we define $\mathcal{N}_\mathrm{b}^\mathrm{e} = \{1, 2, \ldots, N_\mathrm{b}/2\}$

- The objective is to estimate the position $\boldsymbol{p}$ of a target of interest
  - ranging measurements

$$\mathbf{r} = [d_1, d_2, \ldots, d_{N_\mathrm{b}}]^\mathrm{T} + \mathbf{w}$$

  - d_j is the distance between the target and j-th sensing node
  - $\mathbf{w}$ is a multivariate noise with a normal distribution $\mathcal{N}(\mathbf{0}_{N_b \times 1}, \mathrm{diag}(\sigma_1, \ldots, \sigma_{N_\mathrm{b}}))$



PERFORMANCE METRICS FOR NETWORK LOCALIZATION

Fisher Information Inequality

- The fundamental limits of network localization provide performance benchmarks and are essential for designing the network
- Let $\hat{\mathbf{p}}$ be any unbiased estimator of $\boldsymbol{p}$, then under some mild regularity conditions

$$\mathbb{E}\{(\boldsymbol{p}-\hat{\mathbf{p}})(\boldsymbol{p}-\hat{\mathbf{p}})^{\mathrm{T}}\} \succcurlyeq \boldsymbol{J}^{-1}$$

where

$$\boldsymbol{J} \triangleq \sum_{j\in\mathcal{N}_{\mathrm{b}}} \lambda_j \begin{bmatrix} \cos^2(\phi_j) & \cos(\phi_j)\sin(\phi_j) \\ \cos(\phi_j)\sin(\phi_j) & \sin^2(\phi_j) \end{bmatrix}$$

Fisher information matrix (FIM)

- λ_j represents the range information intensity of the j-th node
  - signal-to-noise ratio (SNR) of the signal transmitted by the j-th node, in a synchronized network
- ϕ_j represents the relative angle of the j-th node with respect to the target

Optimal Designs

- Optimal designs in terms of a statistical criterion
  - a sub-field of statistics initiated by Kirstine Smith (1918)

- There are several criteria for assessing the network geometry that can be written as functions of the eigenvalues of $\boldsymbol{J}^{-1}$
  - D-optimality: minimization of $\det(\boldsymbol{J}^{-1})$
  - A-optimality: minimization of $\mathrm{tr}(\boldsymbol{J}^{-1})$
  - E-optimality: minimization of $\mathrm{v}(\boldsymbol{J}^{-1})$, the largest eigenvalue of $\boldsymbol{J}^{-1}$

- We characterize the optimal deployment according to the D-optimality criterion, and its implications for the A-optimality and E-optimality criteria are discussed in our paper

D-optimality

- Minimization of $\det(\boldsymbol{J}^{-1})$ is equivalent to maximizing the FIM determinant

$$\det(\boldsymbol{J}) = \Big(\sum_{j\in\mathcal{N}_\mathrm{b}} \lambda_j \cos^2(\phi_j)\Big)\Big(\sum_{j\in\mathcal{N}_\mathrm{b}} \lambda_j \sin^2(\phi_j)\Big) - \Big(\sum_{j\in\mathcal{N}_\mathrm{b}} \lambda_j \cos(\phi_j)\sin(\phi_j)\Big)^2$$

  – which is upper bounded by the first summand as follow

$$\det(\boldsymbol{J}) \leqslant \bar{\ell}_\mathrm{d} \triangleq (\mathrm{tr}(\boldsymbol{J}) - \Pi)\,\Pi$$

  where

$$\Pi \triangleq \sum_{j\in\mathcal{N}_\mathrm{b}} \lambda_j \sin^2(\phi_j)$$

Perfect Pairing

- For each node $j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}$ consider a node $j' = j + N_{\mathrm{b}}/2$ such that

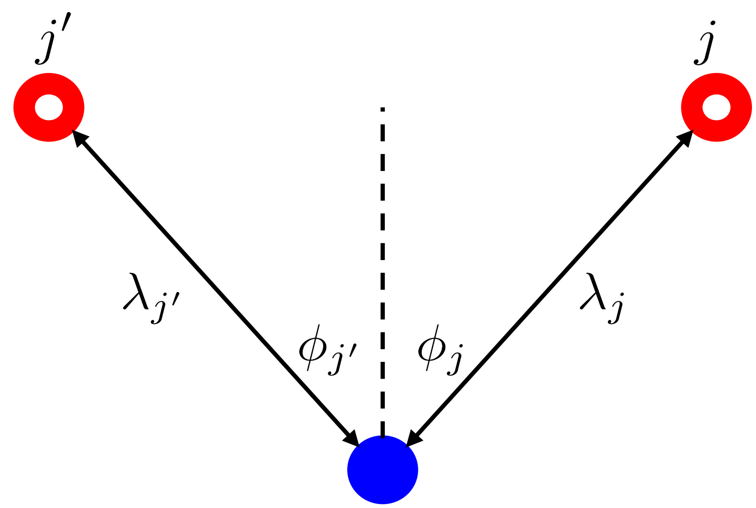
  $\lambda_j = \lambda_{j'}$

  $\phi_j = -\phi_{j'}$

  – in this case, $\det(\boldsymbol{J})$ becomes equal to its upper bound $\bar{\ell}_{\mathrm{d}}$

  – $\bar{\ell}_{\mathrm{d}}$ is maximized if it is possible to set $\phi_j = \pm\pi/4$

  – $\bar{\ell}_{\mathrm{d}}^{*} \triangleq \left(\mathrm{tr}(\boldsymbol{J})\right)^2/4$ is the maximum value
    - the perfect-pairing bound



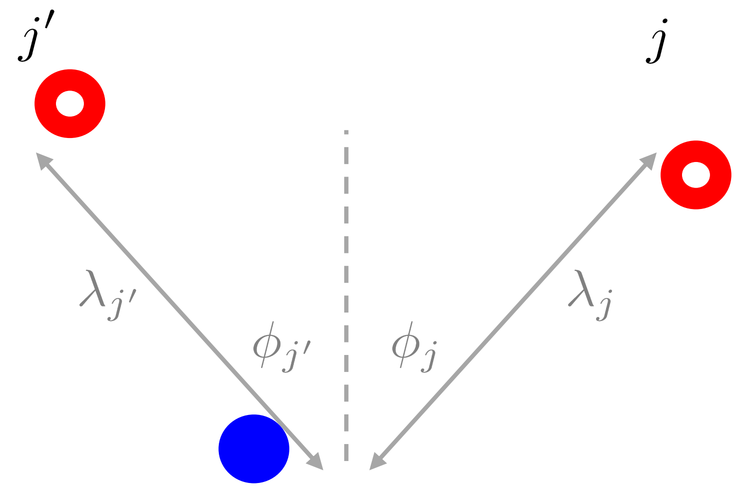
Node Deployment under Position Uncertainty

- The optimal sensor configuration follows the prefect pairing pattern

- In many applications, it is *not* possible to deploy the nodes with perfect pairing
  - indoor positioning systems
    - external disturbances and obstacles
  - Ocean-of-things (OoT)
    - environmental disturbances such as wind and ocean currents
  - Internet-of-Battlefield-Things (IoBT)
    - adversary aims to hamper the localization process of legitimate nodes by forcing them to move from their initial or desired positions

Node Deployment under Position Uncertainty

- Bounded disturbances in the positions of the sensing nodes



RELAXED NODE DEPLOYMENT

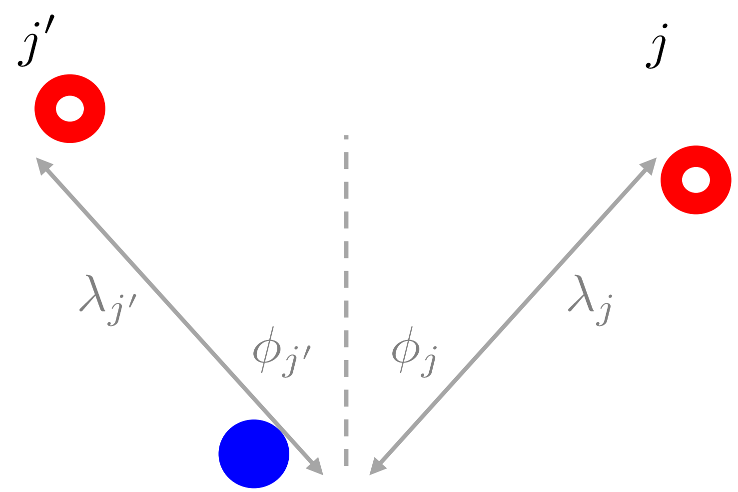
Relaxed Sensor Pairing

- For each node $j \in \mathcal{N}_\mathrm{b}^\mathrm{e}$ consider a node $j' = j + N_\mathrm{b}/2$ such that

$$\lambda_j = \lambda_{j'} + \Delta\lambda_j$$
$$\phi_j = -\phi_{j'} + \Delta\phi_j$$

- Given $\overline{\Delta\lambda} \geqslant 0$ and $\overline{\Delta\phi} \geqslant 0$ a set of nodes are called $(\overline{\Delta\lambda}, \overline{\Delta\phi})$ paired if

$$|\Delta\lambda_j| \leqslant \overline{\Delta\lambda} \qquad \forall j \in \mathcal{N}_\mathrm{b}^\mathrm{e}$$
$$|\Delta\phi_j| \leqslant \overline{\Delta\phi}$$



Bounds on Determinant of FIM

- Given $(\overline{\Delta\lambda}, \overline{\Delta\phi})$ paired nodes
  - characterize upper and lower bounds on $\det(\boldsymbol{J})$
  - serve to identify points or regions in which $\det(\boldsymbol{J})$ is maximized

- Recall
  - $\det(\boldsymbol{J}) \leqslant \bar{\ell}_\mathrm{d} \triangleq (\mathrm{tr}(\boldsymbol{J}) - \Pi)\,\Pi$ where $\Pi \triangleq \sum_{j \in \mathcal{N}_\mathrm{b}} \lambda_j \sin^2(\phi_j)$
  - perfect-pairing bound $\bar{\ell}^*_\mathrm{d} \triangleq \big(\mathrm{tr}(\boldsymbol{J})\big)^2/4$

Bounds on Determinant of FIM

- $(\overline{\Delta\lambda}, \overline{\Delta\phi})$paired nodes: $\underline{\ell}_{\mathrm{d}} \leqslant \det(\boldsymbol{J}) \leqslant \bar{\ell}_{\mathrm{d}}$
  - here $\underline{\ell}_{\mathrm{d}} \triangleq \bar{\ell}_{\mathrm{d}} - \epsilon$ where

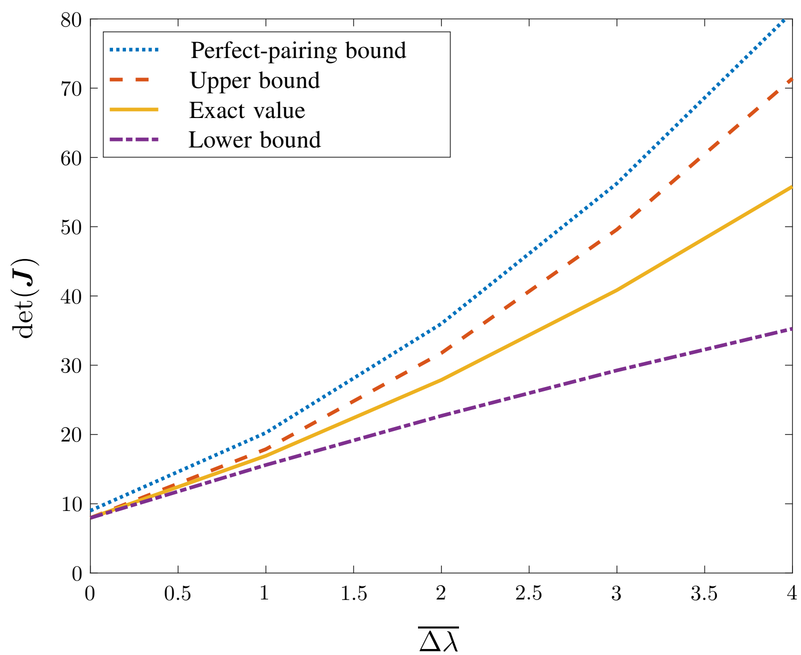
$$\epsilon \triangleq \left(\Big| \sum_{j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}} \lambda_j \sin^2(\Delta\phi_j) \sin(2\phi_j) + \frac{1}{2} \sum_{j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}} \lambda_j \sin(2\Delta\phi_j) \cos(2\phi_j) \Big| + \frac{N_{\mathrm{b}} \overline{\Delta\lambda}}{4} \right)^2$$

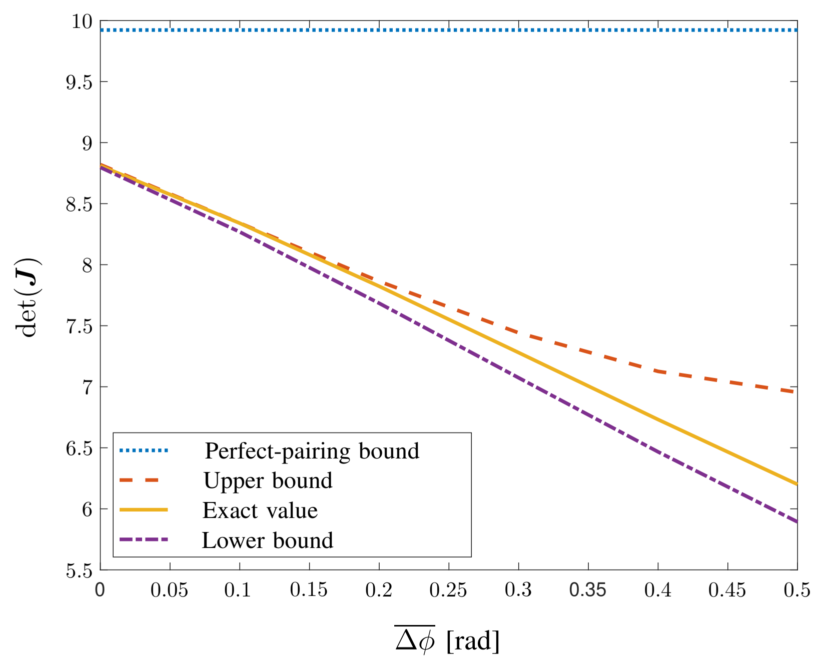
  - furthermore $\bar{\ell}_{\mathrm{d}}^{*} - \bar{\ell}_{\mathrm{d}} \leqslant \delta$

$$\delta \triangleq \left(\Big| \sum_{j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}} \lambda_j \cos^2(\Delta\phi_j) \cos(2\phi_j) + \frac{1}{2} \sum_{j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}} \lambda_j \sin(2\Delta\phi_j) \sin(2\phi_j) \Big| + \frac{N_{\mathrm{b}} \overline{\Delta\lambda}}{4} \right)^2$$

Numerical Results

- The FIM determinant and its upper and lower bounds as functions of
  - the upper bound on the mismatch in the SNR
  - the upper bound on the mismatch in the relative angles





Optimization-based Node Deployment

- Finding the optimal network geometry via optimization
  - D-optimality and the pairing design
  - in many applications, it is not possible to encircle the target with nodes and the range of relative angles for the nodes with respect to the target can be constrained
  - node deployment can be formulated as the following optimization problem

$$
\begin{aligned}
\mathscr{P}_1: \quad & \underset{\boldsymbol{\lambda}^{\mathrm{e}}, \boldsymbol{\phi}^{\mathrm{e}}}{\text{maximize}} && \det(\boldsymbol{J}) \\
& \text{subject to} && 0 \leqslant \lambda_j \leqslant \bar{\lambda}\,, && \forall j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}} \\
& && \iota_1 \leqslant \phi_j \leqslant \iota_2\,, && \forall j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}
\end{aligned}
$$

  - $\iota_1 \in \mathbb{R},\ \iota_2 \in \mathbb{R},\ \bar{\lambda} \in (0, \infty),\ \boldsymbol{\lambda}^{\mathrm{e}} \triangleq [\lambda_1, \lambda_2, \ldots, \lambda_{N_{\mathrm{b}}/2}],\ \boldsymbol{\phi}^{\mathrm{e}} \triangleq [\phi_1, \phi_2, \ldots, \phi_{N_{\mathrm{b}}/2}]$

Optimization-based Node Deployment

$$\mathscr{P}_1: \quad \underset{\boldsymbol{\lambda}^{\mathrm{e}},\boldsymbol{\phi}^{\mathrm{e}}}{\text{maximize}} \quad \det(\boldsymbol{J})$$
$$\text{subject to} \quad 0 \leqslant \lambda_j \leqslant \bar{\lambda} \qquad \forall j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}$$
$$0 \leqslant \phi_j \leqslant \pi/4\,, \qquad \forall j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}}$$

- $\det(\boldsymbol{J})$ is not a straightforward objective for optimization purposes
  – we will find a relevant optimization program, which can be efficiently solved

A Relevant Optimization Program

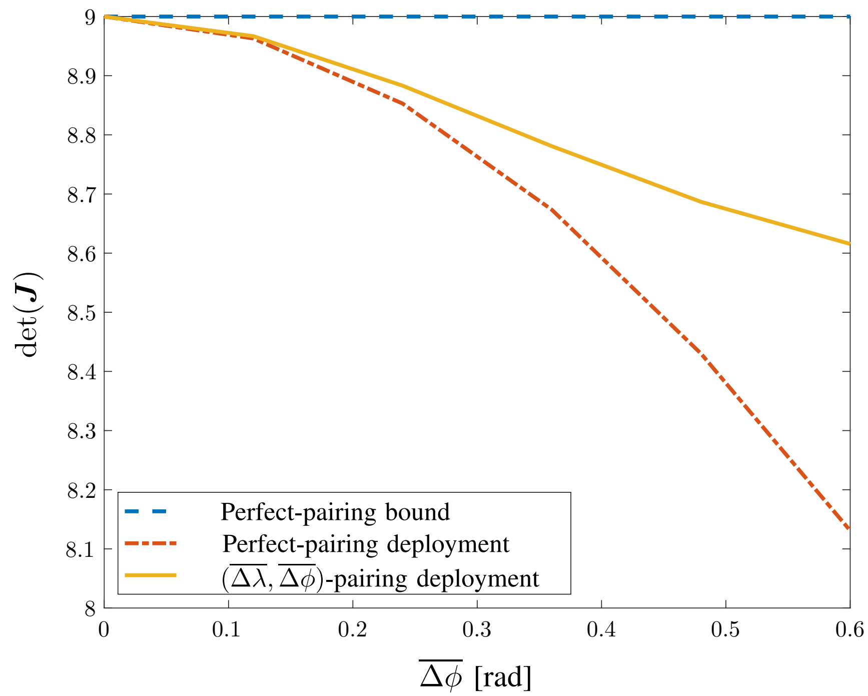
- Minimize ϵ
  - the distance between the lower and upper bounds of $\det(\boldsymbol{J})$
- Minimize δ
  - the distance between the upper bound $\bar{\ell}_d$ and its upper bound $\bar{\ell}_d^* \triangleq \big(\mathrm{tr}(\boldsymbol{J})\big)^2/4$
- Maximize $\bar{\ell}_d^*$

$$
\begin{aligned}
\mathscr{P}_2 : \quad \underset{\boldsymbol{\lambda}^{\mathrm{e}},\boldsymbol{\zeta}^{\mathrm{e}}}{\text{maximize}} \quad & \sum_{j\in\mathcal{N}_{\mathrm{b}}^{\mathrm{e}}} \lambda_j \Big(1-\zeta_j\,\sqrt{\alpha^2+\beta^2}\Big) \\
\text{subject to} \quad & 0 \leqslant \lambda_j \leqslant \bar{\lambda}\,, \qquad \forall j \in \mathcal{N}_{\mathrm{b}}^{\mathrm{e}} \\
& \sin(\tan^{-1}(\alpha/\beta)) \leqslant \zeta_j \leqslant 1\,, \qquad \forall j \in \mathcal{N}_{\mathrm{b}/2}
\end{aligned}
$$

  - where $\zeta_j \triangleq \cos\big(2\phi_j + \tan^{-1}(-\beta/\alpha)\big)$, also α and β are defined in our paper
  - an instance of bilinear programming

Numerical Results

- The FIM determinant as a function of $\overline{\Delta\phi}$
  - fixed SNR



FINAL REMARKS

Final Remarks

- We noticed that uncertainties in the positions of the sensing nodes could deteriorate the performance of the localization networks
  - we developed a framework for optimal node deployment that accounts for uncertainties in the positions of deployed nodes
  - we designed the efficient node deployment algorithm by solving a bilinear program

- We characterized the optimal deployment according to the D-optimality criterion
  - we showed that the proposed optimization-based design achieves an improvement in the D-optimality criterion compared to state-of-the-art methods
  - we also discussed the implications for the A-optimality and E-optimality criteria in our paper

IEEE ICC®

THANK YOU