Learning-enabled optimal quantum control

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Project Overview

Project Objective: Learning-enabled optimal quantum control (OQC) provides a new framework for both learning the dynamics of and controlling quantum systems in a scalable manner. The main objective is to provide engineers with a toolset for efficiently controlling quantum systems with unknown dynamics.

Challenges:
- Unlike classical systems, the state of a quantum system can never be measured or known perfectly.
- The dynamics of closed quantum systems evolve unitarily, but classical system identification strategies do not enforce this constraint.
- Existing quantum process tomography methods scale poorly to large quantum systems.

Project Impact:
- Quantum tomography enabled Hamiltonian Learning (QT-HML) is proposed as an efficient mean for identifying both internal and control Hamiltonians. This is the first quantum tomography based HML algorithm which infers the control dynamics.
- End-to-end optimal control of quantum systems with arbitrary dynamics.

The paradigm of OPC via HML
- QT-HML utilizes experimentally gathered data from quantum experiments to estimate both the internal and control Hamiltonians with high accuracy.
- OQC utilizes the learned model (in computer simulation) to compute a control sequence for the quantum system.
- OQC then provides the optimized control sequence to the quantum system in an open-loop fashion.

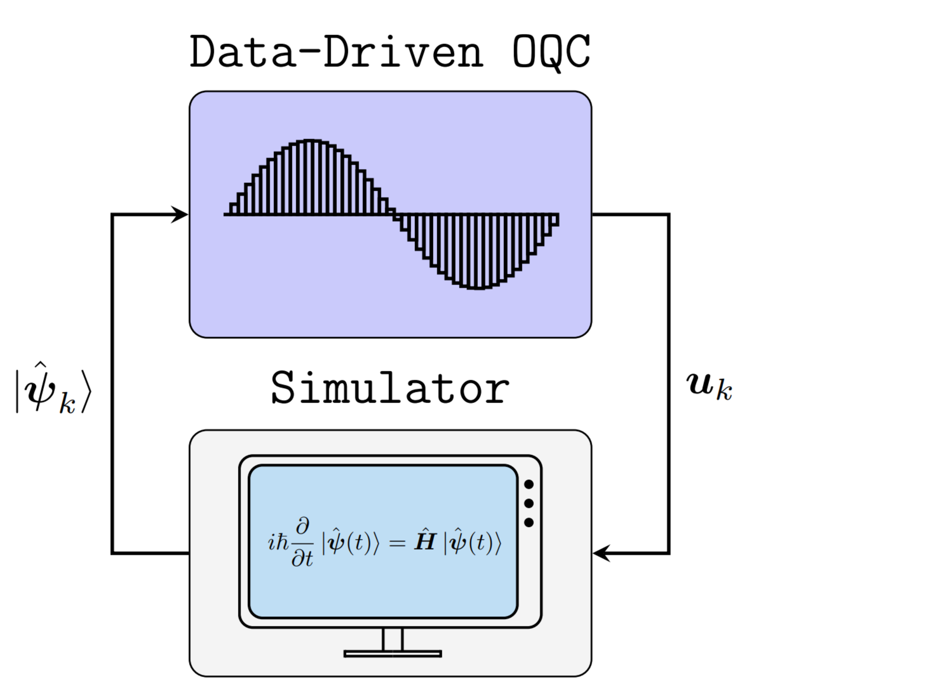


Figure 1: OQC computes a control sequence offline in simulation using the learned Hamiltonian.

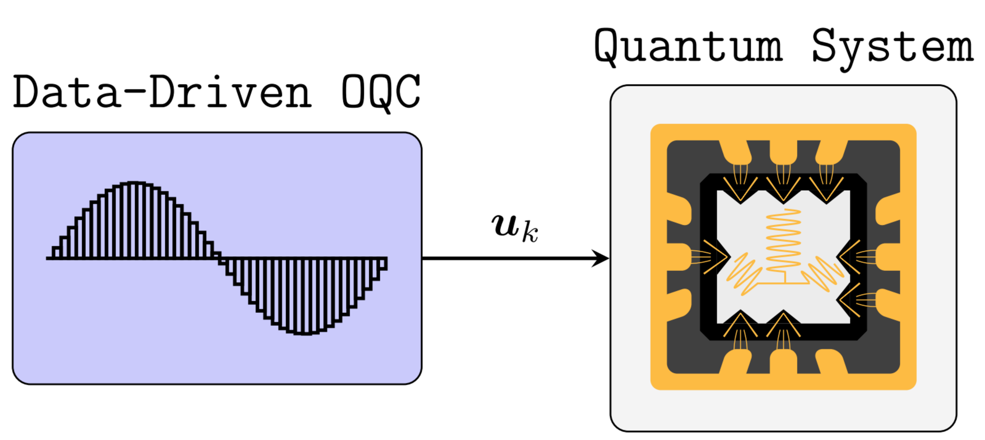


Figure 2: OQC then feeds the optimized control sequence to the system in an open-loop fashion.

Quantum Tomography Enabled Hamiltonian Learning

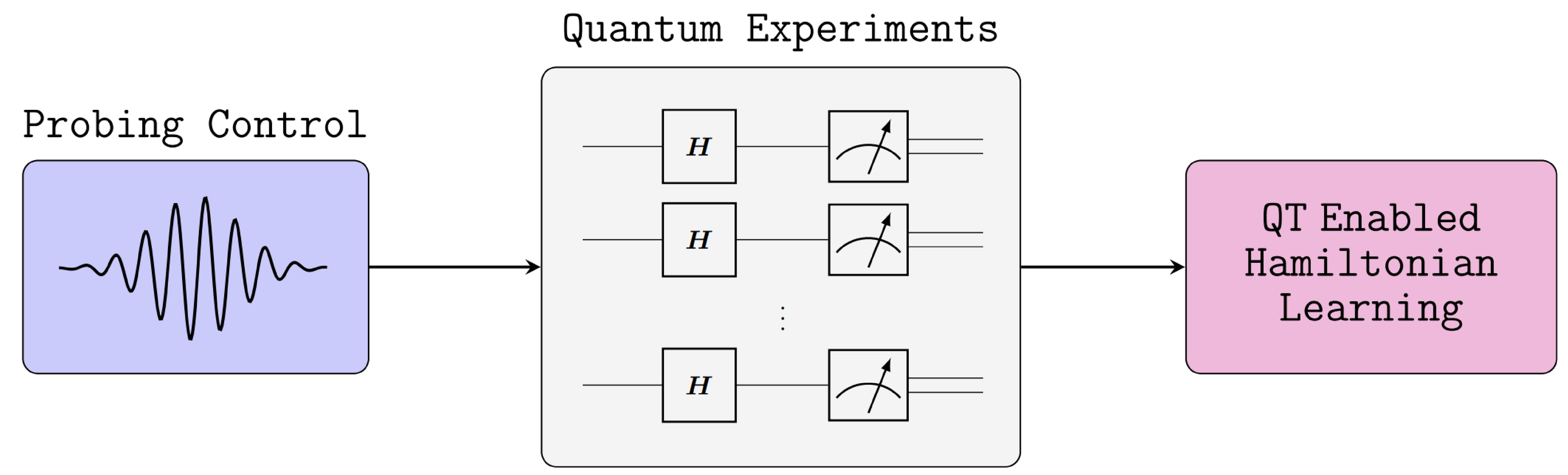


Figure 3: Quantum tomography enabled Hamiltonian learning utilizes a probing control input and quantum experiments to infer the internal and control Hamiltonians of the closed quantum system.

Methodology:
Given a set of initial quantum states

$$\boldsymbol{\Psi}_0 = \begin{bmatrix} |\psi_0^1\rangle & |\psi_0^2\rangle & \dots & |\psi_0^n\rangle \end{bmatrix},$$

quantum experiments are performed with various control inputs and estimates the output states as

$$\hat{\boldsymbol{\Psi}}_{\mathrm{f}} = \begin{bmatrix} |\hat{\psi}_{\mathrm{f}}^1\rangle & |\hat{\psi}_{\mathrm{f}}^2\rangle & \dots & |\hat{\psi}_{\mathrm{f}}^n\rangle \end{bmatrix}.$$

QT-HML learns an appropriate unitary operator which maps the input to output:

$$\hat{U}(t_s) : \boldsymbol{\Psi}_0 \mapsto \hat{\boldsymbol{\Psi}}_{\mathrm{f}},$$

from which it can recover estimates of the system's internal and control Hamiltonians. Figure 3 depicts how the control input, quantum experiments, and QT-HML procedures are connected.

Optimal Quantum Control

Methodology:
Using estimates of the systems internal and control Hamiltonians,

$$\{\hat{\boldsymbol{H}}_0, \hat{\boldsymbol{H}}_1, \dots, \hat{\boldsymbol{H}}_n\}$$

OQC solves the following optimization problem in a closed-loop simulation (Figure 1) to produce an optimal N step discrete-time control sequence to manipulate the state of the system:

$$\begin{aligned} &\underset{\boldsymbol{u}_n : n=1,\dots,N}{\text{minimize}} && \sum_{n=1}^{N} g_n(|\hat{\psi}_n\rangle, \boldsymbol{u}_n) \\ &\text{subject to} && |\hat{\psi}_{n+1}\rangle = e^{-i\hat{\boldsymbol{H}}_n \Delta t}|\hat{\psi}_n\rangle \end{aligned}$$

The optimized control sequence is then provided to the real quantum system in an open-loop fashion (Figure 2).

Numerical Experiments

Part 1: Quantum tomography enabled Hamiltonian learning
QT-HML estimates the ground-truth Hamiltonians

$$\boldsymbol{H}_0 = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix} \quad \text{and} \quad \boldsymbol{H}_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

as

$$\hat{\boldsymbol{H}}_0 = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix} \quad \text{and} \quad \hat{\boldsymbol{H}}_1 = \begin{bmatrix} 0.01 & -i \\ i & -0.01 \end{bmatrix}.$$

Part 2: Optimal quantum control via Hamiltonian learning
Once QT-HML has estimated the system Hamiltonians, OQC computes a control sequence to drive a qubit in the ground state to the excited state. This sequence is then provided to the true quantum system in an open-loop fashion. The optimized control sequence is shown in Figure 4 and the system response in Figure 5.

Figure 4: Control signal produced by optimal quantum control.

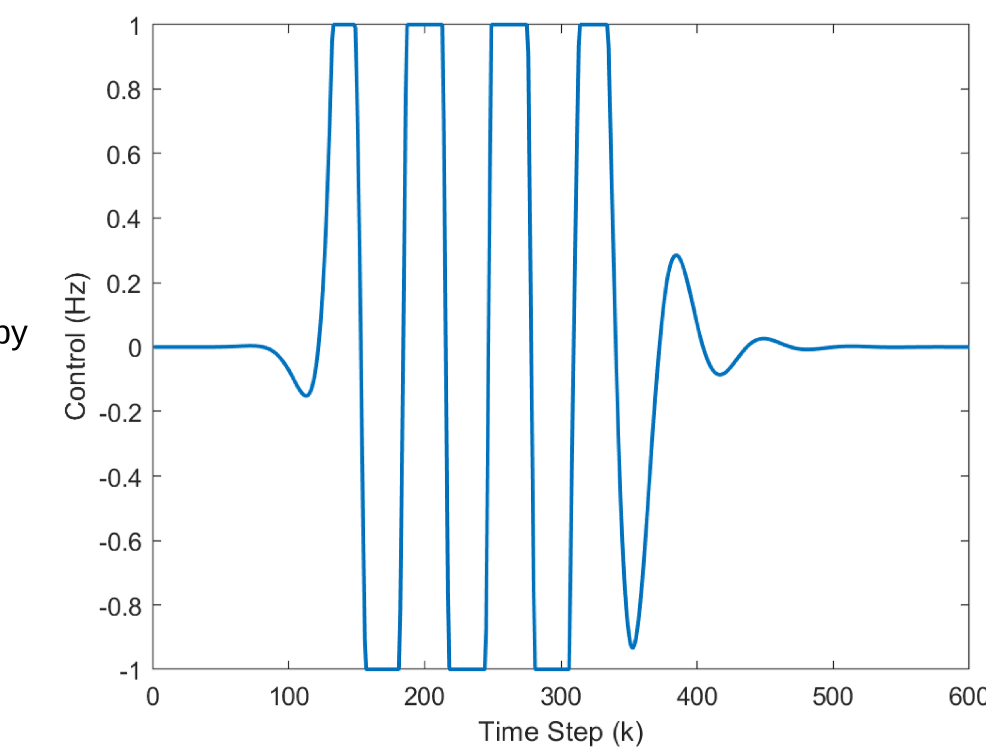
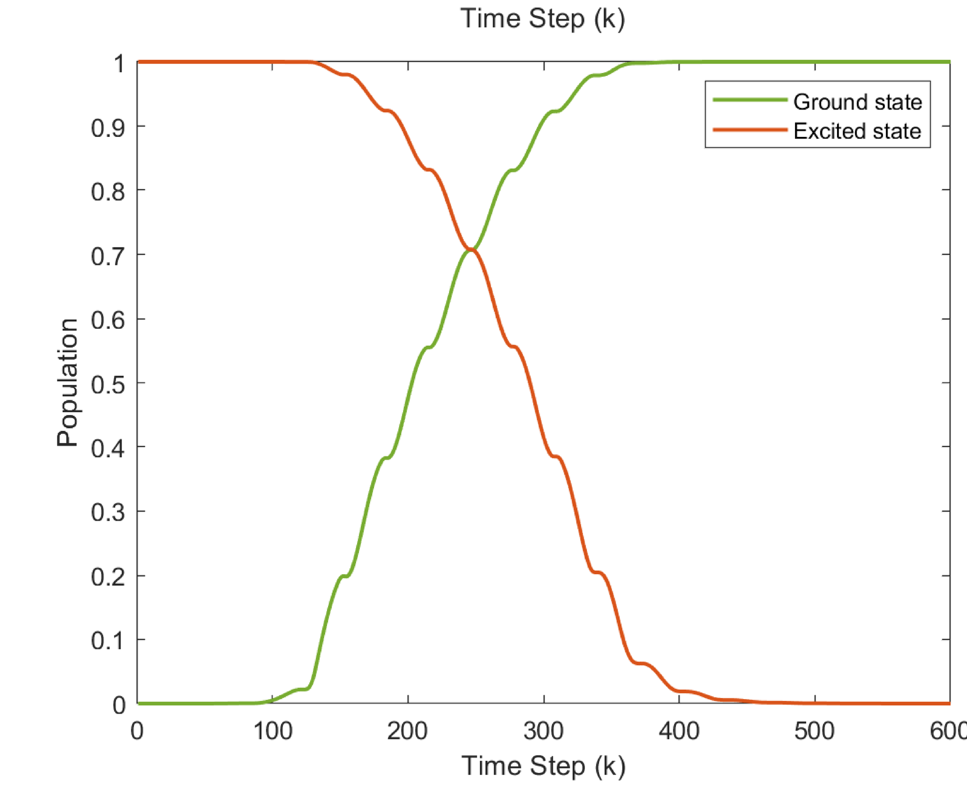


Figure 5: Controlled population levels of the qubit over time.



Conclusion

In this work, a novel QT-HML algorithm is proposed which is both scalable and accurate. Together, QT-HML and OQC form a learning-based means of end-to-end control for quantum systems.