The value of information in event triggering: can we beat the data-rate theorem?

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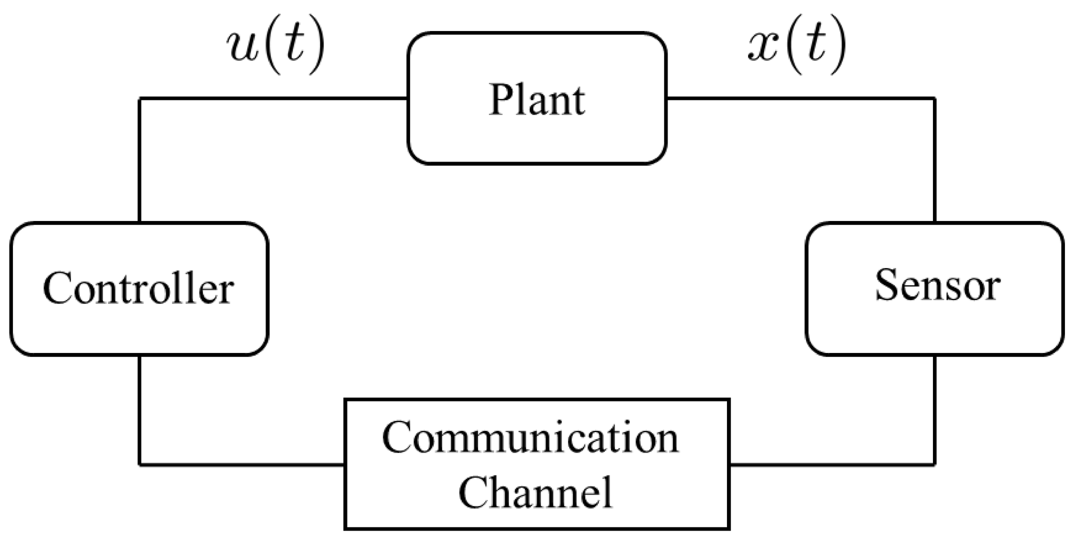
Cyber-physical systems (CPS):

A key aspect of CPS systems is the presence of finite-rate, digital communication channels in the feedback loop.

Networked Control System:

We consider a model where there is a finite-rate, digital communication channel in the feedback loop between the sensor and controller.



The plant has a scalar state with dynamic

$$\dot{x} = Ax(t) + Bu(t),$$

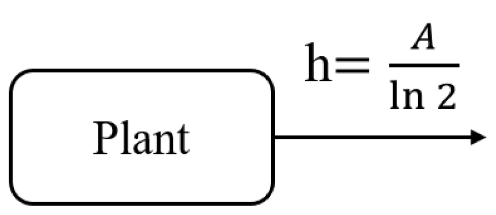
and is unstable, therefore A is a positive real number.

Data-rate theorem:

A system with dynamic

$$\dot{x} = Ax(t) + Bu(t)$$

generates information at entropy rate $h = A/\ln 2$.



Data-rate theorems quantify the effect that communication has on stabilization by stating that the communication rate available in the feedback loop should be at least as large as the entropy rate of the system.

Event-triggered control:

In CPS we need to use distributed resources efficiently

Step 1: --
Step 2: --
Step 3: Bad Dog
Step 4: --
.
.
.

Timing information:

Timing of the triggering events carries information in itself

Let $v(t)$ be a triggering function known to both sensor and controller. Assume the sensor is going to trigger and send a packet to the controller when the absolute value of state estimation error become equal to triggering function namely,

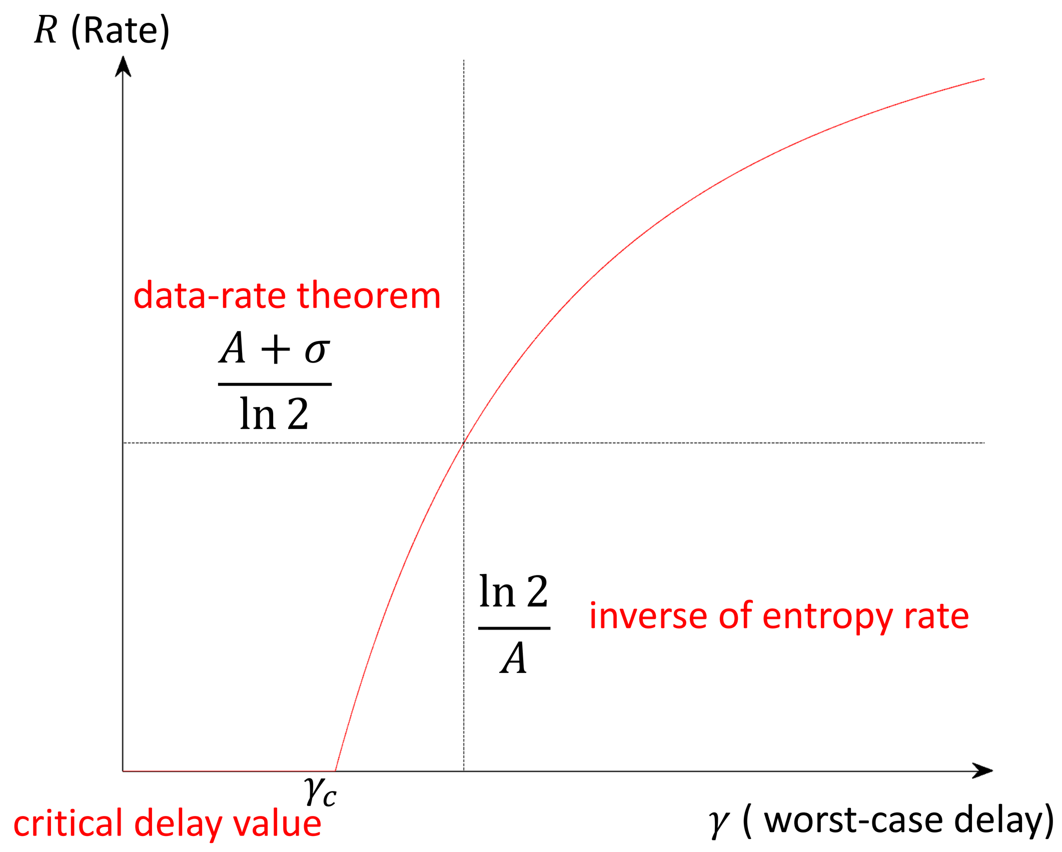
$$|x(t) - \hat{x}(t)| = v(t).$$

If the controller knows the triggering time t, then it can track the state of the system with arbitrary precision by transmitting a single bit corresponding to + or − in the following equation

$$x(t) = \hat{x}(t) \pm v(t).$$

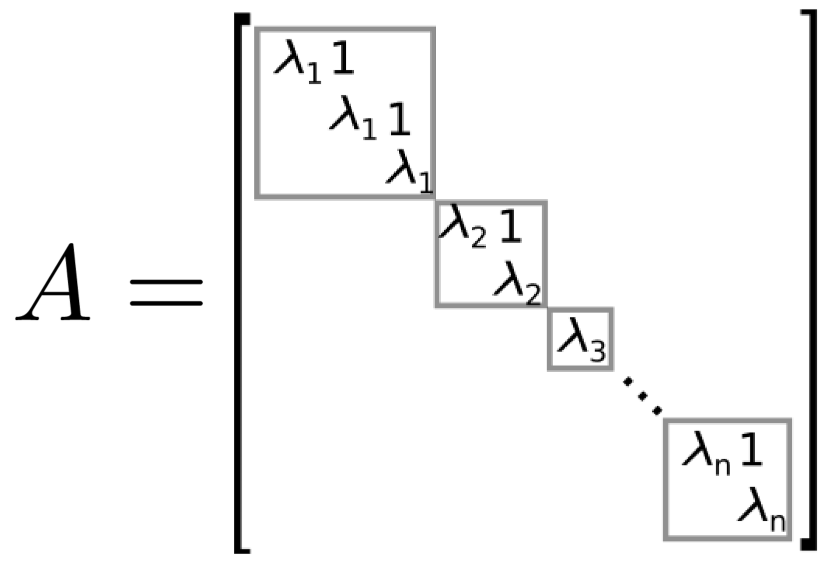
Phase transition:

Assume the event triggering function, $v(t)$, is equal to $e^{-\sigma t}$, where σ is a nonegative real number. Morover, assume the delay in communication channel is upper bounded by γ. Then for sufficiently large σ the below figure illustrates the required transmission rate versus the worst-case delay γ in the communication channel.



Extension to the vector systems:

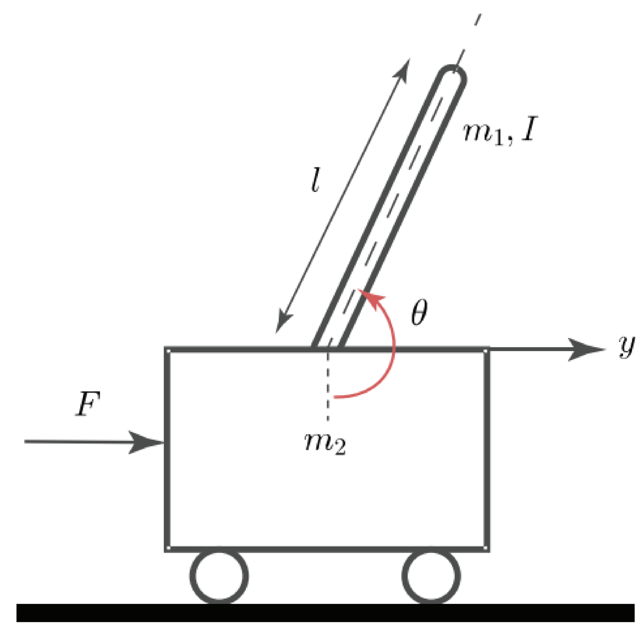
We generalized our results to vector systems building on our analysis of the scalar case. For simplicity of exposition, we assumed that A is equal to its Jordan block decomposition.

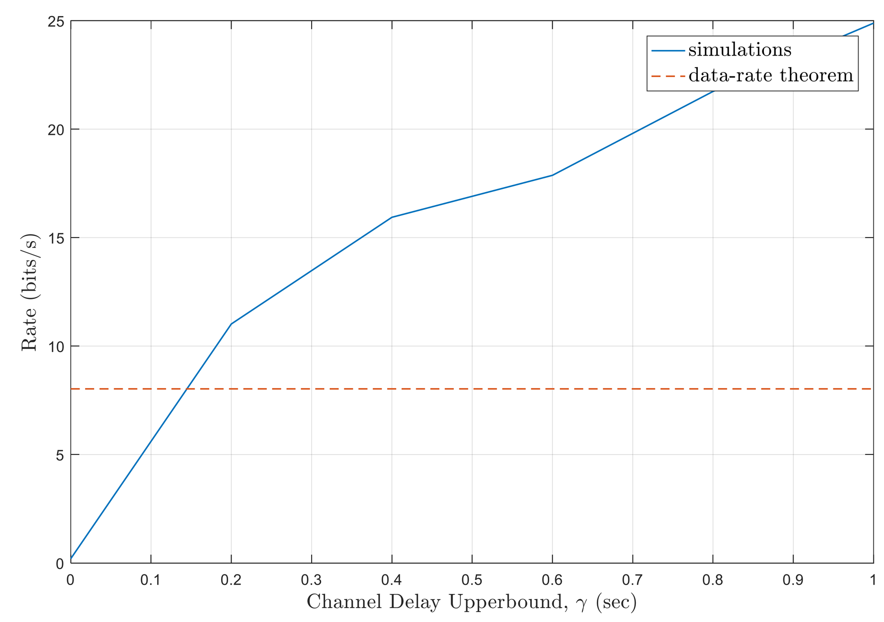


System with additive disturbance:

$$\dot{x} = Ax(t) + Bu(t) + w(t)$$

$$|w(t)| \leq M$$





Reference:

M. J. Khojasteh, P. Tallapragada, J. Cortés, and M. Franceschetti, "The value of timing information in event-triggered control," arXiv preprint arXiv:1609.09594, 2017.

M. J. Khojasteh, M. Hedayatpour, J. Cortés, and M. Franceschetti, "Event-triggered stabilization of disturbed linear systems over digital channels," arXiv preprint arXiv:1801.08704, 2018.