Cognitive-Engined Spectrum-Fragmented Synchronous MC-CDMA Based on Generalized Hadamard Codes

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***Abstract*—In Cognitive Radio (CR), in order to avoid interference in the Primary Users (PU), some subcarriers need to be deactivated. In systems based on Multicarrier Code Division Multiple Access (MC-CDMA), this causes losing orthogonality among different spreading codes, leading to poor Bit Error Rate (BER) performance. The performance of such system can be improved by using Generalized Hadamard Codes (GHC) instead of conventional Hadamard codes which avoids the loss of orthogonality. This is due to the fact that unlike conventional Hadamard codes, GHC are conjectured to exist for any arbitrary length. In this paper, we propose using a novel spreading code, namely, GHC for synchronous MC-CDMA in CR networks. The new spreading codes help the CR systems maintaining the data rate while improving the performance by eliminating the orthogonality loss. Finally, the performance of the MC-CDMA system using GHC is evaluated and it is shown that the proposed system outperform the system with conventional Hadamard codes.**

***Index Terms*—Cognitive Radio (CR), Multi-carrier Code Division Multiple Access (MC-CDMA), Generalized Hadamard Codes (GHC).**

I. Introduction

Rapid development of wireless technology and growing demand for radio spectrum, has led to moving towards a more efficient spectrum utilization procedure than fixed spectrum assignment [1]. Since the licensed spectrum is greatly underutilized, reviewing the policy and seeking more intelligent and flexible communication technology is unavoidable [2]. To increase spectrum efficiency Cognitive Radio (CR) was proposed and recently has been given an increasing attention [2].

CR enables wireless users to optimize their operating parameters adaptively according to the surrounding radio environment. In CR, users without spectrum license (Secondary Users (SU)) are allowed to use the temporarily unused licensed spectrum [3]. Exploiting the spectrum in an opportunistic fashion, CR gives SUs the capability of sensing the available portions of the spectrum, selecting the best available channel, coordinating spectrum access with other users, and vacating the channel as soon as a primary user reclaims the spectrum usage right [3].

Multicarrier Code Division Multiple Access (MC-CDMA) is one of the most robust and spectrally efficient CDMA techniques which is referred to the combination of CDMA and Orthogonal Frequency Division Multiplexing (OFDM), taking the advantages of both techniques, also known as OFDM-CDMA in technical literature [4]. In such a system, an OFDM-based transceiver is used to multiply the code on frequency domain and take advantage of the dense subcarrier spacing of OFDM and the multiple access benefits of CDMA [5].

Bit Error Rate (BER) performance of asynchronous MC-CDMA with Hadamard spreading code has been evaluated in [6]. However, these codes are restricted to code lengths of form $4k$ where k is a positive integer and deactivating redundant subcarriers may also reduce the data rate. To maintain the data rate, conventional MC-CDMA employs a code length larger than the number of active subcarriers and zeros some chips which cause loss of orthogonality. We propose the use of Generalized Hadamard Codes (GHC) to maintain both orthogonality and data rate. The main benefit of GHC is the ability of having any integer length. Hence, by applying it as the spreading code for MC-CDMA, regardless of the number of deactivated subcarriers, the loss of orthogonality among spreading codes can be entirely eliminated.

In this paper, we present and evaluate the performance of a cognitive-engined synchronous MC-CDMA system using GHC in the presence of multi-user interference in Additive White Gaussian Noise (AWGN) and Rayleigh fading channels. Furthermore, the BER performance is compared with systems using conventional Hadamard codes and it is shown that the proposed system outperforms it. In order to validate simulation results, the analytical formulations are numerically plotted and compared with the simulations.

The rest of the paper is organized as follows. In Section II, we describe the method of finding the Generalized Hadamard Matrices (GHM) and GHC. In Section III, performance evaluation of spectrum-fragmented synchronous MC-CDMA transceiver is discussed and closed-form equations for error probability is driven. Analytical and simulation results and discussions are presented in Section IV. Finally, Section V summarizes the paper and presents the conclusion.

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II. Generalized Hadamard Codes

An $n \times n$ matrix, H, over ± 1 is called Hadamard matrix if its rows are orthogonal to each other, in other words

$$HH^T = nI \tag{1}$$

where H^T is the transpose of H, and I is identity matrix of order n. Order n of a Hadamard matrix H is a member of the set $\{1, 2, 4k\}$ where k runs in positive integer numbers [7]. The well-known conjecture due to Hadamard states that there exists a Hadamard matrix of order $n = 4k$, when k runs in positive integer numbers [8].

Generalizing the length of Hadamard-sequences is critical in applications such as CR networks, where the code sequence should have a wide range of lengths since the number of available sub-carriers differs in various conditions. Thus, codes with various lengths, other than $4k$, are needed. We overcoming this bottleneck by expanding the sequence space from bipolar to multi-level.

Definition 1: Consider the set of all $n \times n$ matrices that their elements come from $S \subseteq \mathbb{C} - \{0\}$, and if H is a member of this set then

$$HH^* = Diag(\lambda_1, \ldots, \lambda_n) \tag{2}$$

where H^* is Hermitian of H, and λ_i is a positive number, for $i = 1, \ldots, n$. This set is called *S-GHM*$_n$.

In [10], Akbari and Bahmani applied some combinatorial methods to obtain some example of $\{\pm 1, \pm 2, \pm 3\}$-GHM_n family for $n \leq 50$.

Definition 2: If a matrix is a member of $\{\pm 1, \pm 2, \pm 3\}$-GHM_n for any n, each row of it is called a GHC of length n.

Also there exists a conjecture that the set $\{\pm 1, \pm 2, \pm 3\}$-GHM_n, is not empty for any positive integer n [10]. The unsolved cases up to 31 in ascending order are 11, 17, 25, 27 and 29. By adding ± 4 to $\{\pm 1, \pm 2, \pm 3\}$ the GHMs, and therefore GHCs, can be found easier by combinational methods. The first rows of the GHMs for different lengths between three and fifteen are shown in Table I.

Two major theorems for construction of GHMs can be mentioned as follows [10]

1) If A is the incidence matrix of a 2-(v, k, λ) symmetric design (for further information about symmetric designs and their incidence matrices see [13]), then $AA^T = (k - \lambda)I_v + \lambda U_v$, where I is the $v \times v$ identity matrix and U is the $v \times v$ all ones matrix [13].

Let M be the incidence matrix of a 2-(v, k, λ) symmetric design. By supposing A as a $v \times v$ matrix obtained from replacement of 0 and 1 with s and r in M, respectively, $AA^T = aI_v + bU_v$, where $a = (k - \lambda)(r - s)^2$ and $b = \lambda r^2 + 2(k - \lambda)rs + (v - 2k + \lambda)s^2$ [10]. With this in mind, putting $\{r, s\} = \{1, -2\}$ converts the incidence matrix, N, to a GHM.

2) There is a similar theory to Sylvester method [9] for construction of GHM. Let $S \subseteq \mathbb{C} - \{0\}$ and $H_1, H_2 \in M_n(S)$ ($M_n(S)$ is the set of $n \times n$ matrices with elements that come from S), such that $H_1 H_2^* = H_2 H_1^*$ and $H_1 H_1^* + H_2 H_2^* = Diag(\lambda_1, \ldots, \lambda_n)$, for some positive numbers λ_i, $i = 1, \ldots, n$. Then the matrix

$$\begin{bmatrix} +H_1 & +H_2 \\ -H_2 & +H_1 \end{bmatrix} \tag{3}$$

is a member of $S - GHM_{2n}$ [10].

TABLE I
The first rows of different GHMs used in the proposed method

n	Sequence
3	(+1 -2 -2)
5	(+3 -2 -2 -2 -2)
6	(+1 -2 +1 +1 +1 +1)
7	(-2 -2 +1 -2 +1 +1 -1)
9	(+2 +1 +1 +1 -1 -1 +3 -3 -3)
10	(+2 -1 -1 -1 -1 +1 -1 -1 -1 -1)
11	(+4 +3 -2 +3 +3 +3 -2 -2 -2 +3 -2)
13	(+1 +1 -1 +1 -1 -1 -1 +1 +1 +1 -1 +1)
14	(+2 -1 -1 -1 -1 -1 -1 -1 -1 +1 -1 +1 +1 +1)
15	(+2 +2 +2 -1 -1 -1 -1 +2 +2 +2 +2 -1 -1 -1 -1)

Considering the fact that all the members of *S-GHM*$_n$ could not be constructed by the above theorems, for finding some members combinatorial methods and some tricks are used.

Example 1 (GHM of length 21): Considering the first theorem mentioned above we use 2-$(21, 5, 1)$ design to construct a member of $\{-2, 1\} - GHM_{21}$.

Example 2 (GHM of length 22): One of the corollaries of the mentioned theorem in [10] says that if H_1, H_2 are circulant matrix satisfying the condition of second theorem and R is back-diagonal identity matrix, then

$$\begin{bmatrix} H_1 & H_2 \\ -H_2 R & RH_1 \end{bmatrix} \tag{4}$$

is a member of $S - GHM_{2n}$. But this corollary is used with a little trick to construct GHM of length 22. Let $H_1 = Circ\{+1 + 1 - 1 + 1 + 1 + 1 - 1 - 1 - 1 + 1 - 1\}$ and $H_2 = Circ\{-1 + 1 - 2 + 1 + 1 + 1 - 2 - 2 - 2 + 1 - 2\}$, the inner product of the rows of H_1 and H_2 is not equal to zero but with the help of above corollary H_1 and H_2 give us a member of GHM_{22}.

III. Performance Evaluation

In this section, we provide an analytical performance evaluation of the MC-CDMA-based cognitive radio transceivers with GHC and use this analytical result in our numerical analysis to validate the simulation results. The transmitted signal of the kth user is shown as

$$s_{Tx}^{(k)}(t) = \sum_{n=1}^{N} d^{(k)} c_n^{(k)} p_n(t) e^{j2\pi f_n t}, \quad 0 < t < T_b \tag{5}$$

where $d^{(k)} \in \{-1, +1\}$ is the data symbol of the kth user, $c_n^{(k)}$ is the nth chip of the kth user's normalized code, and $p_n(t)$ is $\frac{1}{\sqrt{T_b}}$ for $t \in [0, T_b]$, otherwise zero, in which T_b is the bit duration. Each chip of the users' code is set to zero whenever a Primary Users (PU) on that chip is detected active at the channel sensing procedure. We assume that SUs sense the channel in collaboration with each other and the result of this cooperative spectrum sensing is used by all of them. Even with this cooperation, there is some probability that CR users cannot detect the presence of PUs (miss-detection) or falsely declare a sub-channel as active (false alarm) [11]. The received data which is the combination of all SUs signal after considering noise, Rayleigh fading channel effect and miss-detection of some PUs can be presented as

$$s_{Rx}(t) = \sum_{k=1}^{K}\sum_{n=1}^{N} \beta_n^{(k)} d^{(k)} c_n^{(k)} p_n(t) e^{j2\pi f_n t} + n(t) + \sum_{i\in\Lambda} s_i(t) e^{j2\pi f_i t}, \quad 0 < t < T_b \tag{6}$$

where $\beta_n^{(k)}$ stands for channel fading of the nth frequency chip for the kth user which has unity variance, Λ represents all the subcarriers having miss-detection of PUs in the sensing procedure, $s_i(t)$ is the miss-detected PU signal which is supposed to have Gaussian distribution. The matched filter output for the first user in the nth sub-channel can be calculated as

$$\begin{aligned} r_n =& \frac{1}{\sqrt{T_b}} \int_0^{T_b} \beta_n^{(1)*} c_n^{(1)} e^{-j2\pi f_n t} s_{Rx}(t)\, dt \\ =& \frac{1}{T_b} \int_0^{T_b} \beta_n^{(1)*} c_n^{(1)} e^{-j2\pi f_n t} \left\{ \sum_{k=1}^{K}\sum_{\tilde{n}=1}^{N} \beta_{\tilde{n}}^{(k)} d^{(k)} c_{\tilde{n}}^{(k)} e^{j2\pi f_{\tilde{n}} t} \right\} dt \\ &+ \frac{1}{\sqrt{T_b}} \int_0^{T_b} \beta_n^{(1)*} c_n^{(1)} e^{-j2\pi f_n t} \{ n(t) + \sum_{i\in\Lambda} s_i(t) e^{j2\pi f_i t} \} dt \end{aligned} \tag{7}$$

Now, let us define $u(n) = 1$ for $n \in \Lambda$ and otherwise zero. Hence, r_n can be manipulated in the following form

$$\begin{aligned} r_n =& \left|\beta_n^{(1)}\right|^2 d^1 c_n^{(1)^2} + \sum_{k=2}^{K} \beta_n^{(1)*} \beta_n^{(k)} d^{(k)} c_n^{(1)} c_n^{(k)} \\ &+ \frac{1}{\sqrt{T_b}} \int_0^{T_b} \beta_n^{(1)*} c_n^{(1)} \{ \bar{n}(t) + u(n) s_n(t) \} dt \end{aligned} \tag{8}$$

where $\bar{n}(t) = n(t) e^{-j2\pi f_n t}$ has the same distribution as $n(t)$. For the decision procedure, since the modulation is BPSK we should calculate the real part of the received signal as follows

$$\begin{aligned} Re\{r_n\} =& \left|\beta_n^{(1)}\right|^2 d^{(1)} c_n^{(1)^2} + \sum_{k=2}^{K} \Big(Re\{\beta_n^{(1)}\} Re\{\beta_n^{(k)}\} \\ &+ Im\{\beta_n^{(1)}\} Im\{\beta_n^{(k)}\} \Big) d^{(k)} c_n^{(1)} c_n^{(k)} \\ &+ \frac{1}{\sqrt{T_b}} \int_0^{T_b} \Big(Re\{\beta_n^{(1)}\} c_n^{(1)} Re\{\bar{n}(t) + u(n) s_n(t)\} \\ &+ Im\{\beta_n^{(1)}\} c_n^{(1)} Im\{\bar{n}(t) + u(n) s_n(t)\} \Big) dt \end{aligned} \tag{9}$$

Thus, the decision variable for the first user would be

$$\begin{aligned} R =& \sum_{n=1}^{N} Re\{r_n\} = \sum_{n=1}^{N} \Bigg\{ \left|\beta_n^{(1)}\right|^2 d^{(1)} c_n^{(1)^2} + \sum_{k=2}^{K} (Re\{\beta_n^{(1)}\} Re\{\beta_n^{(k)}\} \\ &+ Im\{\beta_n^{(1)}\} Im\{\beta_n^{(k)}\}) d^{(k)} c_n^{(1)} c_n^{(k)} \\ &+ \frac{1}{\sqrt{T_b}} \int_0^{T_b} \Big(Re\{\beta_n^{(1)}\} c_n^{(1)} Re\{\bar{n}(t)\} \\ &+ Im\{\beta_n^{(1)}\} c_n^{(1)} Im\{\bar{n}(t)\} \Big) dt \Bigg\} \\ &+ \sum_{n\in\Lambda} \frac{1}{\sqrt{T_b}} \int_0^{T_b} \Big(Re\{\beta_n^{(1)}\} c_n^{(1)} Re\{s_n(t)\} \\ &+ Im\{\beta_n^{(1)}\} c_n^{(1)} Im\{s_n(t)\} \Big) dt \end{aligned} \tag{10}$$

we can break the decision variable R, to the following components

$$R = R_s + R_{MUI} + R_n + R_{GI} \tag{11}$$

where R_s is the desired signal that can be written as

$$R_s = d^{(1)} \sum_{n=1}^{N} \left|\beta_n^{(1)}\right|^2 c_n^{(1)^2} \tag{12}$$

R_{MUI} is the multi user interference which is calculated as

$$\begin{aligned} R_{MUI} =& \sum_{n=1}^{N}\sum_{k=2}^{K} \big((Re\{\beta_n^{(1)}\} Re\{\beta_n^{(k)}\} \\ &+ Im\{\beta_n^{(1)}\} Im\{\beta_n^{(k)}\}) d^{(k)} c_n^{(1)} c_n^{(k)} \big) \end{aligned} \tag{13}$$

R_n is the received impairment caused by noise, calculated in the following expression

$$\begin{aligned} R_n =& \frac{1}{\sqrt{T_b}} \sum_{n=1}^{N} \int_0^{T_b} c_n^{(1)} (Re\{\beta_n^{(1)}\} Re\{\bar{n}(t)\} \\ &+ Im\{\beta_n^{(1)}\} Im\{\bar{n}(t)\}) dt \end{aligned} \tag{14}$$

and finally, R_{GI} is the interference caused by miss-detected PUs which is calculated as follows

$$\begin{aligned} R_{GI} =& \sum_{n\in\Lambda} \frac{1}{\sqrt{T_b}} \int_0^{T_b} c_n^{(1)} (Re\{\beta_n^{(1)}\} Re\{s_n(t)\} \\ &+ Im\{\beta_n^{(1)}\} Im\{s_n(t)\}) dt \end{aligned} \tag{15}$$

It can be proven that all of the terms in summations represented in (12)-(15) are uncorrelated. Hence, according to the central limit theorem, for large N and K we can approximate R_s, R_{MUI} and R_n to be independent Gaussian random variables [14]. Therefore, we get

$$R_s \sim N\left(d^1 \sum_{n=1}^{N} c_n^{(1)^2}, \sum_{n=1}^{N} c_n^{(1)^4} \right) \tag{16}$$

$$R_{MUI} \sim N\left(0, \sum_{n=1}^{N}\sum_{k=2}^{K} c_n^{(1)^2} c_n^{(k)^2} / 2 \right) \tag{17}$$

$$R_n \sim N\left(0, \frac{1}{2}\sigma_n^2 \sum_{n=1}^{N} c_n^{(1)^2} \right) \tag{18}$$

R_{GI} is a zero mean random variable whose variance is calculated from (15) as fallows

$$\sigma_{R_{GI}}^2 = \frac{1}{2}\sigma_s^2 \sum_{n\in\Lambda} c_n^{(1)^2} \tag{19}$$

Hence, using (16)-(19) and considering $d_1 = 1$ the mean and the variance of the first user's decision variable can be presented in the following forms

$$E(R) = \sum_{n=1}^{N} c_n^{(1)^2} \tag{20}$$

$$\begin{aligned} Var(R) = &\sum_{n=1}^{N} c_n^{(1)^4} + \sum_{n=1}^{N}\sum_{k=2}^{K} |c_n^{(1)}|^2 |c_n^{(k)}|^2/2 \\ &+ \frac{1}{2}\sigma_n^2 \sum_{n=1}^{N} c_n^{(1)^2} + \frac{1}{2}\sigma_s^2 \sum_{n\in\Lambda} c_n^{(1)^2} \end{aligned} \tag{21}$$

Assume l to be the number of subcarriers in which PUs are miss-detected thereby causing interference and m to be the number of subcarriers that are thought to be occupied by the PUs, thus, $|\Lambda| = l$. Conditioning on the parameters m and l and supposing that $d_1 = 1$ the conditional error probability can be calculated from (20) and (21) using the following expression

$$P_{e|l,m} = P(R < 0 \ |d_1 = 1) = Q\left(\frac{\sum_{n=1}^{N} c_n^{(1)^2}}{\sqrt{Var(R)}}\right) \tag{22}$$

We define P_d as the detection probability, P_{fa} as the false alarm probability for each sub-channel, $P(H^1)$ as the occupation probability and $P(H^0) = 1 - P(H^1)$ as the probability of a sub-channel being idle. Therefore, the probability of miss-detection is $(1-P_d)P(H^1)$ and probability of detecting a channel to be active is $P(H^1)P_d + P(H^0)P_{fa}$. Hence, the probability of l subcarriers being miss-detected and m subcarriers being detected active is

$$\begin{aligned} P_{l,m} = &\binom{N}{m}\binom{N-m}{l}\{(1-P_d)P(H^1)\}^l \\ &\times \{P(H^1)P_d + P(H^0)P_{fa}\}^m \\ &\times \{P(H^0)(1-P_{fa})\}^{N-m-l} \end{aligned} \tag{23}$$

By averaging (22) on m and l, P_e can be calculated as follows

$$\begin{aligned} P_e = &\sum_{m=0}^{N-1}\sum_{l=0}^{N-m}\binom{N}{m}\binom{N-m}{l}\{(1-P_d)P(H^1)\}^l \\ &\times \{P(H^1)P_d + P(H^0)P_{fa}\}^m \{P(H^0)(1-P_{fa})\}^{N-m-l} \\ &\times Q\left(\frac{\sum_{n=1}^{N} c_n^{(1)^2}}{\sqrt{Var(R)}}\right) \end{aligned} \tag{24}$$

The error probability obtained in (24) is used in the next section in numerical analysis to validate the simulation results.

IV. Analytical and Simulation Results

The performance of the proposed MC-CDMA system using GHC in the presence of the multi user interference for AWGN and Rayleigh fading channel models are simulated using MATLAB in order to compare with the analytical results. In simulations, the FFT size is 64, N is 31 and detection, false alarm, and occupation probabilities are set to 0.95, 0.05 and 0.1, respectively. Figure 1 presents the BER versus SNR curve for two different number of users in Rayleigh fading and AWGN channels. It compares the performance of conventional Hadamard codes with the proposed GHC in AWGN channel as well. It is noteworthy to mention that in this figure "*Gen.*" stands for Generalized and "*Con.*" stands for conventional which refers to the type of Hadamard codes used. The solid lines represent the analytically obtained BER values and dots stand for simulation results with the same parameters. It is observable that the proposed GHC outperforms the conventional Hadamard codes in AWGN channel. Since the GHC used provide perfect orthogonality for each number of available sub-channels, the BER does not vary for different number of users in AWGN channels. However, in Rayleigh fading channel, as the number of users increases, the interference due to fading is increased which cause degradation in BER performance. Various curves in the figure imply an excellent agreement between the analytical and simulation results.

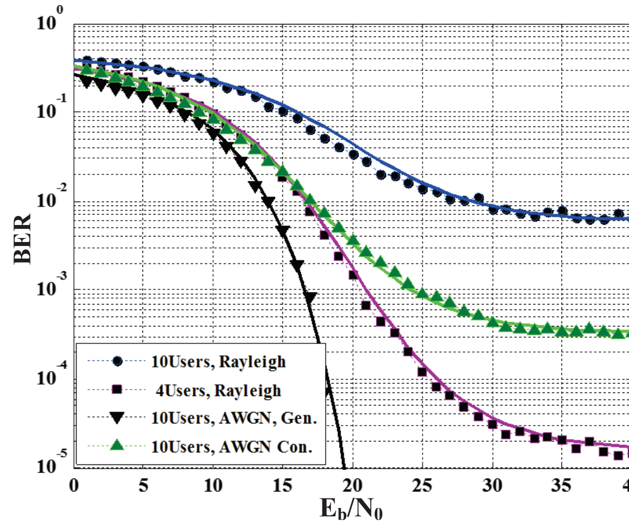

Fig. 1. BER performance versus SNR of the GHC in AWGN and Rayleigh fading channel (solid lines: analytical, dots: simulation)

V. Conclusion

We propose using a novel spreading code which is called GHC for MC-CDMA in CR networks. The new spreading code help the system to maintain the same data rate while improving the performance by eliminating the orthogonality loss. The performance of the MC-CDMA system using GHC is evaluated and compared with conventional systems showing better BER performance of the proposed system.

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