Stabilizing a linear system using phone calls

Mohammad Javad Khojasteh, Massimo Franceschetti, Gireeja Ranade

***Abstract*— We consider the problem of stabilizing an undisturbed, scalar, linear system over a "timing" channel, namely a channel where information is communicated through the timestamps of the transmitted symbols. Each transmitted symbol is received at the controller subject to some to random delay. The sensor can encode messages in the holding times between successive transmissions and the controller must decode them from the inter-reception times of successive symbols. This set-up is analogous to a telephone system where a transmitter signals a phone call to the receiver through a "ring" and, after a random time required to establish the connection, is aware of the "ring" being received. We show that for the state to converge to zero in probability, the *timing capacity* of the channel should be at least as large as the entropy rate of the system. In the case the symbol delays are exponentially distributed, we show a tight sufficient condition using a decoding strategy that successively refines the estimate of the decoded message every time a new symbol is received. These results extend our previous work on estimation over the timing channel to stabilization.**

I. INTRODUCTION

A wide range of Cyber-Physical Systems (CPS) can be modeled as networked control systems where the feedback loop is closed over a communication channel [1]. In these systems event-triggering control strategies have become popular due to their efficient usage of the communication and computation resources. In this case, it has been shown that the timing of the triggering events carries information that can be used for stabilization [2]–[9]. Motivated by this observation, the goal of this paper is to quatnify the value of the timing information from an information-theoretic perspective, when this is used for control. We consider a specific communication channel in the loop — a *timing channel*. Here, information is communicated through the timestamps of the symbols transmitted over the channel; the "time" is carrying the message.

We consider stabilization of a scalar, undisturbed, continuous-time, unstable, linear system over a timing channel and rely on the information-theoretic notion of *timing capacity* of the channel, namely the amount of information that can be encoded using time stamps [10]–[21]. In this setting, the sensor can communicate with the controller by choosing the timestamps at which symbols from a unitary alphabet are transmitted. The controller receives each transmitted symbol after a *random delay* is added to the time-stamp. When the feedback loop is closed over a communication channel, data-rate theorems quantify the impact of the communication channel on the ability to stabilize the system. Roughly speaking, these theorems state that to achieve stabilization the communication rate available in the feedback loop should be at least as large as the intrinsic entropy rate of the system, expressed by the sum of its unstable modes [22]–[25]. We prove the following data-rate theorem for stabilization over a timing channel. For the state to converge to zero in probability, the timing capacity of the channel should be at least as large as the entropy rate of the system. Conversely, in the case the random delays are exponentially distributed, we show that when this inequality is satisfied, a random coding strategy along with a successively refining decoder can be used to drive the state to zero in probability. In a previous work [26], we obtained analogous results for estimation over a timing channel, and here we extend these results to stabilization.

The books [22], [23] and the surveys [24], [25] provide detailed discussions of data-rate theorems and related results. A portion of the literature studied stabilization over "bit-pipe channels," where a rate-limited, possibly time-varying and erasure-prone communication channel is present in the feedback loop [27]–[31]. For noisy channels, Tatikonda and Mitter [32] showed that for undisturbed linear systems, to let the state to converge to zero a.s., the Shannon capacity of the channel should be larger than the entropy rate of the system. Matveev and Savkin [33] showed that this condition is also sufficient for discrete memoryless channels, but a stronger condition is required in the presence of disturbances, namely the zero-error capacity of the channel must be larger than the entropy rate of the system [34]. Nair [35] derived a similar information-theoretic result in a deterministic setting. Sahai and Mitter [36] considered the moment-stabilization over noisy channels and in the presence of system disturbances of bounded support, and provided a data-rate theorem in terms of the anytime capacity of the channel. They showed that to keep the mth moment of the state bounded, the aytime capacity of order m should be larger than the entropy rate of the system. The anytime capacity has been further investigated in [37]–[40]. Matveev and Savkin [23, Chapter 8] have also introduced a weak notion of stabilty in probability, requiring the state to be bounded with probability $(1-\epsilon)$ by a constant that diverges as $\epsilon \to 0$, and showed that in this case it is possible stabilize linear systems with bounded disturbances over noisy channels provided that the Shannon capacity of the channel is larger than the entropy rate of the system. The various results, along with our contribution, are summarized in Table I.

The main point that can be drawn from the above results is that the relevant capacity notion for stabilization over a

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M. J. Khojasteh and M. Franceschetti are with the Department of Electrical and Computer Engineering, University of California, San Diego. {mkhojasteh,massimo}@ucsd.edu

G. Ranade is with Microsoft Research and the EECS department at the University of California, Berkeley. Some of this work was performed while at the Simons Institute for the Theory of Computing. gireeja@eecs.berkeley.edu

communication channel critically depends on the notion of stability and on the system's model. From the system's perspective, our set-up is closest to the one in [27], [32], [33], as there are no disturbances and the objective is to drive the state to zero. Our convergence in probability provides a stronger necessary condition, but a weaker sufficient condition than the one in these works. Our convergence in probability is also significantly stronger than the notion of stabilization in probability [23, Ch. 8].

Parallel work in control theory has investigated the possibility of stabilizing linear systems using timing information. One primary focus of the emerging paradigm of event-triggered control [41]–[44] has been on minimizing the number of transmissions while simultaneously ensuring the control objective [7], [45], [46]. In this context, the works in [3], [5]–[9] pointed out that the timing of the state-dependent triggering events carries information that can be used for stabilization. It has been shown that the amount of timing information is sensitive to the delay in the communication channel. While for small delay stabilization can be achieved with by transmitting data payload (physical data) at a rate arbitrarily close to zero, for large values of the delay this is not the case, and the data payload transmission rate must be increased [6], [9]. In this paper we extend these results from an information-theoretic perspective, as we explicitly quantify the value of the timing information, independent of any transmission strategy. While here we restrict to transmitting symbols from a unitary alphabet, it would also be of interest to develop "mixed" strategies, using both timing information and physical data transmitted over a larger alphabet. Other important research directions left open for future investigation regard generalizations to vector systems and the study of systems with disturbances. In the latter case, it is likely that usage of stronger notions of capacity, or weaker notions of stability, will be necessary.

Due to space constraints, proofs are omitted for brevity and appear in an extended version of the paper [47].

A. Notation

Let $X^n = (X_1, \cdots, X_n)$ denote a vector of random variables and let $x^n = (x_1, \cdots, x_n)$ denote its realization. If the $X_1, \cdots, X_n$ are independent and identically distributed (i.i.d), then we refer to a generic $X_i \in X^n$ by X and skip the subscript i. We use $\log$ and $\ln$ to denote the logarithms base 2 and base e respectively. We use $H(X)$ to denote the Shannon entropy of a discrete random variable X and $h(X)$ to denote the differential entropy of a continuous random variable X. Further, we use $I(X, Y)$ for the mutual information between random variables X and Y. We write $X_n \xrightarrow{P} X$ if X_n converges in probability to X. Similarly, we will write $X_n \xrightarrow{a.s.} X$ if X_n converges almost surely to X.

II. System and Channel Model

We consider the networked control system depicted in Fig. 1. The system dynamics are described by a scalar, continuous-time, noiseless, linear time-invariant (LTI) system

$$\dot{X}(t) = aX(t) + bU(t), \tag{1}$$

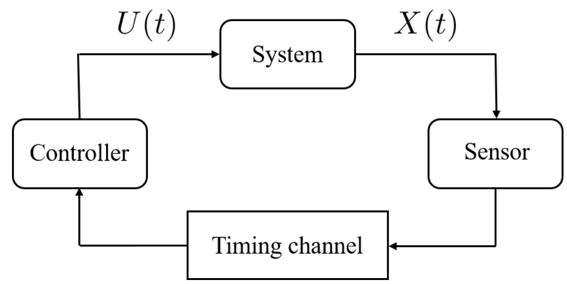

Fig. 1. Model of a networked control system where the feedback loop is closed over a timining channel.

where $X(t) \in \mathbb{R}$ and $U(t) \in \mathbb{R}$ are the system state and the control input respectively. The constants $a, b \in \mathbb{R}$ such that $a > 0$ and $b \neq 0$. The initial state, $X(0)$, is random and is drawn from a distribution of bounded differential entropy and bounded support, namely $h(X(0)) < \infty$ and $|X(0)| < L$, where L is known to both the sensor and the controller. Conditioned on the realization of $x(0)$, the system evolves deterministically. Both controller and sensor have knowledge of the system dynamics in (1). We assume the sensor can measure the state of the system with infinite precision, and the controller can apply the control input to the system with infinite precision and with zero delay.

The sensor is connected to the controller through a timing channel (the telephone signaling channel defined in [10]) and acts as an encoder by choosing to transmit the symbol ♠ at some given times times to the controller. This symbol is delivered to the controller after a random delay, and the sensor receives an instantaneous, causal acknowledgement when the symbol is delivered. The causal acknowledgment can be obtained without assuming an additional communication channel in the feedback loop. Provided that the control input changes at each reception time, the sensor can compute the control input from the current state, and detect whether the previous symbol has been received. Alternatively, the controller can directly signal the acknowledgement to the sensor by applying a control input to the system that excites a specific frequency of the state each time a symbol has been received. These strategies are known in the literature as "acknowledgemebt through the control input" [9], [32], [36].

The encoder uses a "waiting time" to encode information, i.e., after the ith ♠ has been received by the controller, the sensor waits for W_{i+1} seconds to transmit the next symbol. We assume that the channel is initialized with a symbol received at $t = 0$ and that the causal acknowledgement is not used to choose the waiting times, but only to avoid queuing, ensuring that the next symbol is sent after the previous one has been received [10], [13].

The encoding process we described is analogous to that of a telephone system where a transmitter signals a phone call to the receiver through a "ring" and, after a random time required to establish the connection, is aware of the "ring" being received. Communication between transmitter and receiver can then occur without any vocal exchange, but encoding messages in the waiting times between consecutive

TABLE I

CAPACITY NOTIONS USED TO DERIVE DATA-RATE THEOREMS IN THE LITERATURE UNDER DIFFERENT NOTIONS OF STABILITY, CHANNEL TYPES, AND SYSTEM DISTURBANCES.

Work	Disturbance	Channel	Stability condition	Capacity
[27]	NO	Bit-pipe	$\lvert X(t)\rvert \to 0$ a.s.	Shannon
[32], [33]	NO	DMC	$\lvert X(t)\rvert \to 0$ a.s.	Shannon
[34]	bounded	DMC	$\mathbb{P}(\sup_t \lvert X(t)\rvert < \infty) = 1$	Zero-Error
[23, Ch. 8]	bounded	DMC	$\mathbb{P}(\sup_t \lvert X(t)\rvert < K_\epsilon) > 1-\epsilon$	Shannon
[36]	bounded	DMC	$\sup_t \mathbb{E}(\lvert X(t)\rvert^m) < \infty$	Anytime
[28]	unbounded	Bit-Pipe	$\sup_t \mathbb{E}(\lvert X(t)\rvert^2) < \infty$	Shannon
[30], [31], [40]	unbounded	Var. Bit-pipe	$\sup_t \mathbb{E}(\lvert X(t)\rvert^m) < \infty$	Anytime
This paper	NO	Timing	$\lvert X(t)\rvert \xrightarrow{P} 0$	Timing

calls.

Let D_i be the inter-reception time between two consecutive symbols, i.e.,

$$D_i = W_i + S_i, \tag{2}$$

where $\{S_i\}$ are random delays that are assumed to be i.i.d. Fig. 2 provides an example of the timing channel in action.

In our stability analysis, we assume the use of a capacity achieving random codebook, namely the holding times $\{W_i\}$ used to encode any given message are i.i.d. and also independent of the random delays $\{S_i\}$. This assumption is made for analytical convenience, and does not change the capacity of the communication channel.

We assume that at each reception of the nth ♠ the decoder will use the set of n timestamps to decode, and as $n \to \infty$ the decoder refines its estimate of the decoded state. The reception time of the nth symbol is given by $\mathcal{T}_n = \sum_{i=1}^n D_i$. Our objective is to stabilize the system by driving the state to zero in probability, i.e. we want $|X(t)| \xrightarrow{P} 0$ as $t \to \infty$.

The following definitions are derived from [10], incorporating our random coding assumption.

Definition 1: A (n, M, T, δ)-i.i.d.-timing code for the telephone signaling channel consists of a codebook of M codewords $\{(w_i^{(m)}, i = 1, \ldots, n), m = 1 \ldots M\}$, where the symbols in each codeword are picked i.i.d. from a common distribution as well as a decoder, which upon observation of $(D_1, \ldots, D_n)$ selects the correct transmitted codeword with probability at least $1-\delta$. Moreover, the codebook is such that the expected random arrival time of the nth symbol, given by $\mathcal{T}_n = \sum_{i=1}^n D_i$, is not larger than T,

$$\mathbb{E}\left[\mathcal{T}_n\right] \leq T.$$

Definition 2: The rate of an (n, M, T, δ)-i.i.d.-timing code is

$$R = (\log M)/T.$$

Definition 3: The timing capacity C of the telephone signaling channel is the supremum of the achievable rates, namely the largest R such that for every $\gamma > 0$ there exists a sequence of $(n, M_n, T_n, \delta_{T_n})$-iid-timing codes that satisfy

$$\frac{\log M_n}{T_n} > R - \gamma,$$

and $\delta_{T_n} \to 0$ as $n \to \infty$.

The capacity definition in [10] is not restricted to random coding. However, the following result [10, Theorem 8] applies to our random coding set-up, since the capacity in [10] is achieved by random codes.

Theorem 1 (Anantharam and Verdú): The timing capacity of the telephone signaling channel is given by

$$C = \sup_{\chi > 0} \sup_{\substack{W \geq 0 \\ \mathbb{E}[W] \leq \chi}} \frac{I(W; W+S)}{\mathbb{E}[S] + \chi},$$

and if S is exponentially distributed then

$$C = \frac{1}{e\mathbb{E}[S]} \quad \text{[nats/sec]}. \tag{3}$$

III. MAIN RESULTS

To derive necessary and sufficient conditions for the stabilization of the feedback loop system depicted in Fig. 1, we rely on previous results we obtained for an estimation problem over the timing channel [26]. We then derive a necessary condition for stabilization showing that if the state of the closed-loop system $|X(t)| \xrightarrow{P} 0$ as $t \to \infty$, then the error in our estimation problem must also tend to zero in probability. Similarly, we obtain a sufficient condition for stabilization showing that if our estimation error tends to zero in probability, then we can also design a controller such that in closed loop $|X(t)| \xrightarrow{P} 0$. The main idea at the basis of our argument is that in the absence of disturbances all is needed to drive the state to zero is to reliably communicate the initial condition to the controller. This idea has been exploited [48] before [49], and we cast it here in the framework of the timing channel.

The proof of the necessary condition employs a rate-distortion argument to compute a lower bound on the minimum number of bits required to represent the state up to any given accuracy, and this leads to a corresponding lower bound on the required timing capacity of the channel. As a consequence, our necessary condition holds for any source and channel coding strategies adopted by the sensor, and for any strategy adopted by the controller to generate the control input.

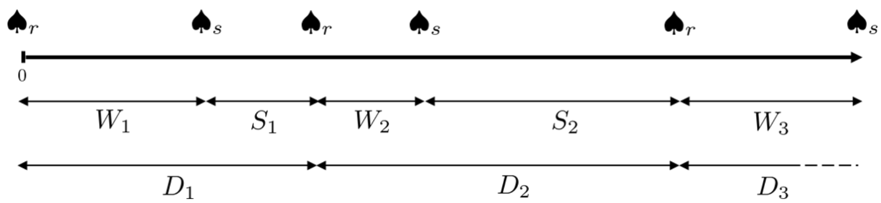

Fig. 2. The timing channel. Subscripts s and r are used to denote sent and received symbols, respectively.

The proof of the the sufficient condition relies on the capacity-achieving code construction in [10]. In addition, in our design we use a maximum likelihood decoder at any time a symbol is received to successively refine the controller's estimate of the initial condition. This approach is similar to the one in [48].

A. The estimation problem

We consider the estimation problem depicted in Fig. 3. By letting $b = 0$ in (1) we obtain the open-loop equation

$$\dot{X}_e(t) = aX_e(t). \tag{4}$$

The objective now is to obtain an estimate of the state $\hat{X}_e(t_n)$, given the reception of n symbols over the telephone signaling channel, such that $|X_e(t_n) - \hat{X}_e(t_n)| \xrightarrow{P} 0$ as $n \to \infty$, at any sequence of *estimation times* t_n such that

$$1 < \lim_{n\to\infty} \frac{t_n}{\mathbb{E}[\mathcal{T}_n]} \leq \Gamma. \tag{5}$$

As in the stabilization problem, we assume that the encoder has causal knowledge of the reception times via acknowledgements through the system as depicted in Fig. 3.

The two following theorems are the building blocks for our stabilization results and appear in [26]. First, we provide a necessary rate for the state estimation problem.

Theorem 2: Consider the estimation problem depicted in Fig. 3 with system dynamics (4). Consider transmitting n symbols over the telephone signaling channel (2), and a sequence of estimation times satisfying (5). If $|X_e(t_n) - \hat{X}_e(t_n)| \xrightarrow{P} 0$, then

$$I(W; W + S) \geq a\ \Gamma\ \mathbb{E}[W + S] \quad \text{[nats]},$$

and consequently

$$C \geq \Gamma a \quad \text{[nats/sec]}.$$

The next theorem provides a sufficient condition for convergence of the state estimation error to zero in probability at any sequence of estimation times t_n given in (5), in the case of exponentially distributed delays.

Theorem 3: Consider the estimation problem depicted in Fig. 3 with system dynamics (4). Consider transmitting n symbols over the telephone signaling channel (2). Assume $\{S_i\}$ are drawn i.i.d. from exponential distribution with mean $\mathbb{E}[S]$. If the capacity of the timing channel is at least

$$C > a\Gamma \quad \text{[nats/sec]},$$

then for any sequence of times $\{t_n\}$ that satisfies (5), we can compute an estimate $\hat{X}_e(t_n)$ such that as $n \to \infty$, we have

$$|X_e(t_n) - \hat{X}_e(t_n)| \xrightarrow{P} 0.$$

B. The stabilization problem

We now turn to consider the stabilization problem. Our first lemma states that if in closed-loop we are able to drive the state to zero in probability, then in open-loop we are also able to estimate the state with vanishing error in probability.

Lemma 1: Consider stabilization of the closed-loop system (1) and estimation of the open-loop system (4) over the timing channel (2). If there exists a controller such that $|X(t)| \xrightarrow{P} 0$ as $t \to \infty$, in closed-loop, then there exists an estimator such that $|X_e(t) - \hat{X}_e(t)| \xrightarrow{P} 0$ as $t \to \infty$, in open-loop.

The next theorem provides a necessary rate for the stabilization problem.

Theorem 4: Consider the stabilization of the system (1). If $|X(t)| \xrightarrow{P} 0$ as $t \to \infty$, then

$$I(W; W + S) \geq a\ \mathbb{E}[W + S] \quad \text{[nats]},$$

and consequently

$$C \geq a \quad \text{[nats/sec]}.$$

Our next lemma strengthens our estimation results, stating that it is enough for the state estimation error to converge to zero in probability as $n \to \infty$ along any sequence of estimation times $\{t_n\}$ satisfying (5), to ensure it converges to zero at all times $t \to \infty$.

Lemma 2: Consider estimation of the system (4) over the timing channel (2). If there exists $\Gamma_0 > 1$ such that along the sequence of estimation times $t_n = \Gamma_0 \mathbb{E}(\mathcal{T}_n)$ we have $|X_e(t_n) - \hat{X}_e(t_n)| \xrightarrow{P} 0$ as $n \to \infty$, then for all $t \to \infty$ we also have $|X_e(t) - \hat{X}_e(t)| \xrightarrow{P} 0$.

The next lemma states that if we are able to estimate the state with vanishing error in probability, then we are also able to drive the state to zero in probability.

Lemma 3: Consider stabilization of the closed-loop system (1) and estimation of the open-loop system (4) over the timing channel (2). If there exists an estimator such that $|X_e(t) - \hat{X}_e(t)| \xrightarrow{P} 0$ as $t \to \infty$, in open-loop, then there exists a controller such that $|X(t)| \xrightarrow{P} 0$ as $t \to \infty$, in closed-loop.

The final theorem provides a sufficient condition for convergence of the state to zero in probability in the case of exponentially distributed delays.

Theorem 5: Consider the stabilization of the system (1). Assume $\{S_i\}$ are drawn i.i.d. from exponential distribution

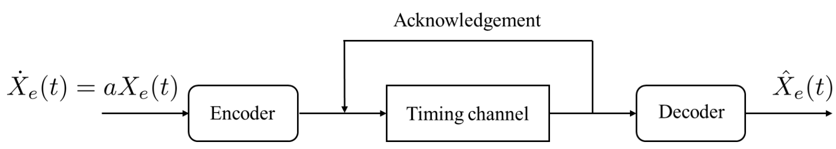

Fig. 3. The estimation problem.

with mean $\mathbb{E}[S]$. If the capacity of the timing channel is at least

$$C > a \quad \text{[nats/sec]},$$

then $|X(t)| \xrightarrow{P} 0$ as $t \to \infty$.

IV. Comparison with previous works

We now discuss some related work in more detail. First, Tatikonda and Mitter in [32] considered the problem of stabilization of the discrete-time version of the system in (1) over an erasure channel. In their model, at each time step of the system's evolution the sensor transmits a packet of I bits to the controller and this is delivered with probability $1-\mu$, or it is dropped with probability μ. It is shown that a necessary condition for $X(k) \xrightarrow{a.s} 0$ is

$$(1-\mu)I \geq \log a \quad \text{[bits/sec]}. \tag{6}$$

From Theorem 4 we obtain the following necessary condition for $X(t) \xrightarrow{P} 0$:

$$\frac{I(W; W+S)}{\mathbb{E}[W+S]} \geq a \qquad \text{[nats/sec]}. \tag{7}$$

We now compare (6) and (7). The rate of expansion of the state space of the continuous system in open loop is a nats per unit time, while for the discrete system is $\log a$ bits per unit time. Accordingly, in the case of (7) the controller must receive at least $a\mathbb{E}[W+S]$ nats representing the initial state during a time interval of average length $\mathbb{E}[W+S]$. Similarly, in the case of (6) the controller must receive at least $\log a/(1-\mu)$ bits representing the initial state over a time interval whose average length corresponds to the average number of trials before the first successful reception

$$(1-\mu)\sum_{k=0}^{\infty}(k+1)\mu^k = \frac{1}{1-\mu}.$$

The works [2], [3], [5]–[9] use event-triggerred policies that exploit timing information for stabilization over a digital communication channel. However, most of these event triggering policies encode information over time in a specific state-dependent fashion. Our framework generalizes this idea to provide a fundamental limit on the rate at which information can be encoded in time in Theorem 4. Theorem 5 achieves this limit in the case of exponentially distributed delays.

The authors of [3], [5] consider stabilization over a zero-delay digital communication channel. They showed that in this case using event triggering it is possible to achieve stabilization with any positive transmission rate, thus implicitly using the information in the timing. For channels without delay, an alternative policy to the one in [3], [5] could be to transmit a single symbol at time equal to any bijective mapping of $x(0)$ into point of non-negative reals. For example, we could transmit $\spadesuit$ at time $t = \tan^{-1}(x(0))$ for $t \in [0, \pi]$. The reception of the symbol would reveal the initial state exactly, and the system could stabilized.

The authors of [6] showed that when delay is positive, but sufficiently small, a triggering policy can still achieve stabilization with any positive transmission rate. However, as the delay increases past a critical threshold, the timing information becomes so much out-of-date that the transmission rate must begin to increase. In our case, since the capacity of our timing channel depends on the distribution of the delay, we may also expect that a large value of the capacity, corresponding to a small average delay, would allow for stabilization to occur using only timing information. Indeed, when delays are distributed exponentially, from (3) and Theorems 4 and 5 it follows that as longs as the expected value of delay is

$$\mathbb{E}[S] < \frac{1}{ea},$$

it is possible to stabilize the system by using only the implicit timing information. The system is not stabilizable using only implicit timing information if the expected value of the delay becomes larger than $(ea)^{-1}$.

V. Conclusions

Recently, it has been shown that event triggering policies, encoding information over time in a state-dependent fashion can exploit timing information for stabilization over a digital communication channel. In a more general framework, this paper studied the fundamental limitation of using timing information for stabilization, independent of any transmission strategy. We showed that for stabilization of an undisturbed scalar linear system over a channel with a unitary alphabet, the timing capacity should be at least as large as the entropy rate of the system. In addition, in the case of exponentially distributed delay, we provided a tight sufficient condition too. Important open problems for future research include the effect of system disturbances, understanding the combination of timing information and packets with payload, and extensions to vector systems.

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