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# DAREK

## Distance Aware Error for Kolmogorov Networks

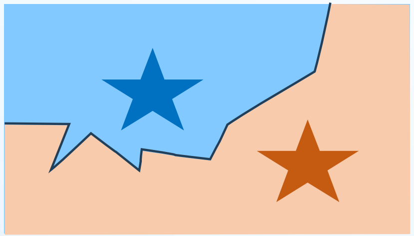
Masoud Ataei

Co-authors: Mohammad Javad Khojasteh and Vikas Dhiman

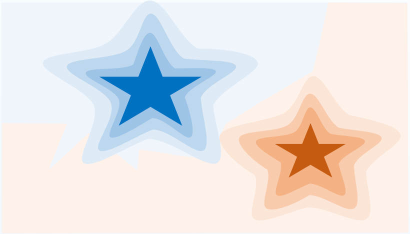
# Motivation

Safe critical applications

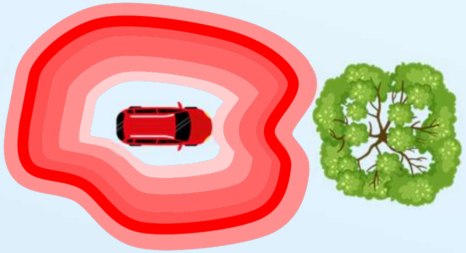
Over confidence model



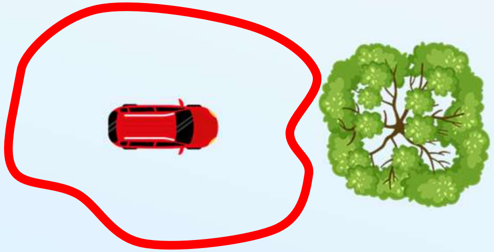
Uncertainty bounded



Probabilistic safety



Worst-case bounded



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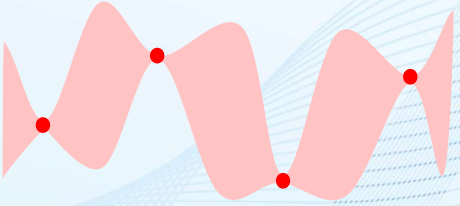
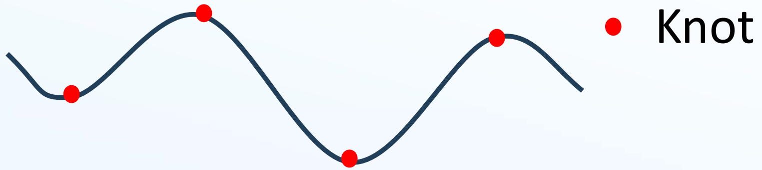


# Introduction

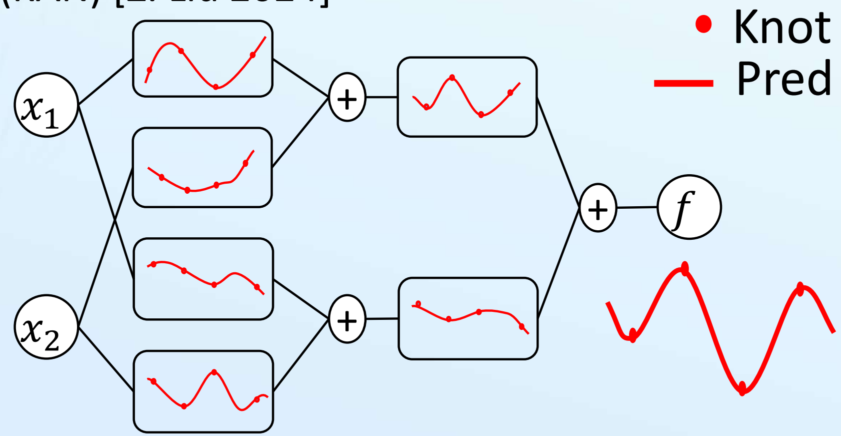
Kolmogorov Arnold Representation Theorem (KAT) [A. N. Kolmogorov 1957]

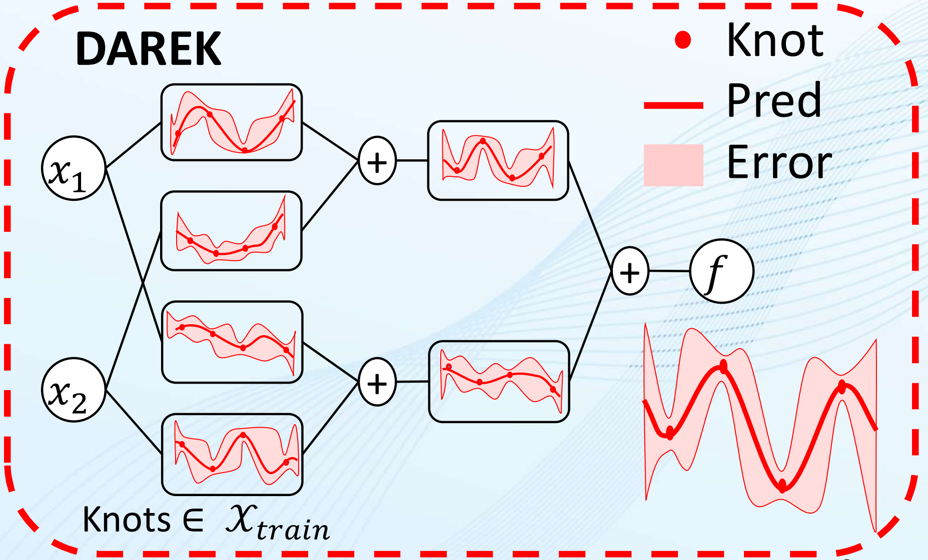
$$f(x_1, x_2, \ldots, x_n) = \sum_{q=1}^{2n+1} \Phi_q\left(\sum_{p=1}^{n} \psi_{p,q}(x_p)\right)$$

Spline

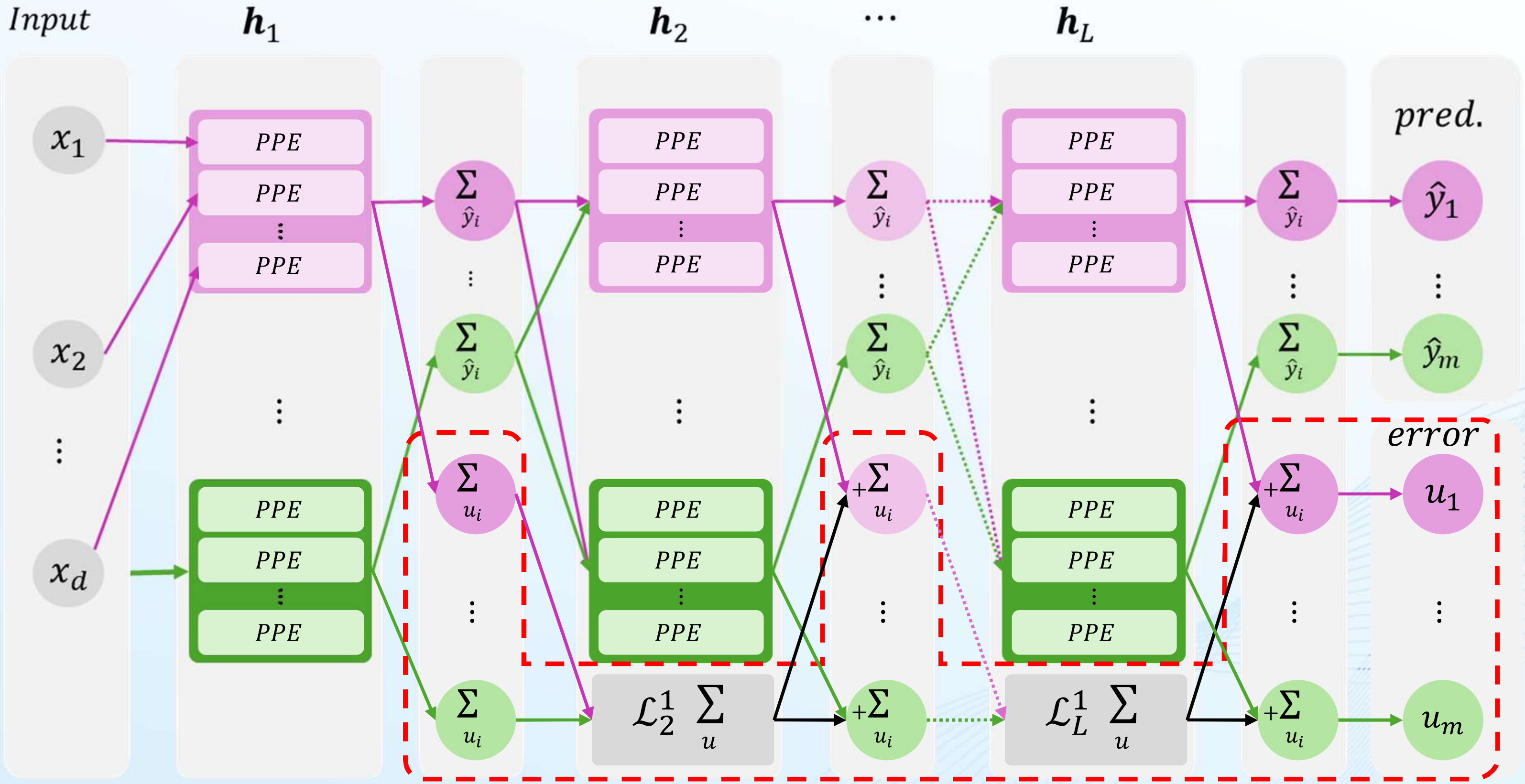



Kolmogorov Arnold Networks (KAN) [Z. Liu 2024]



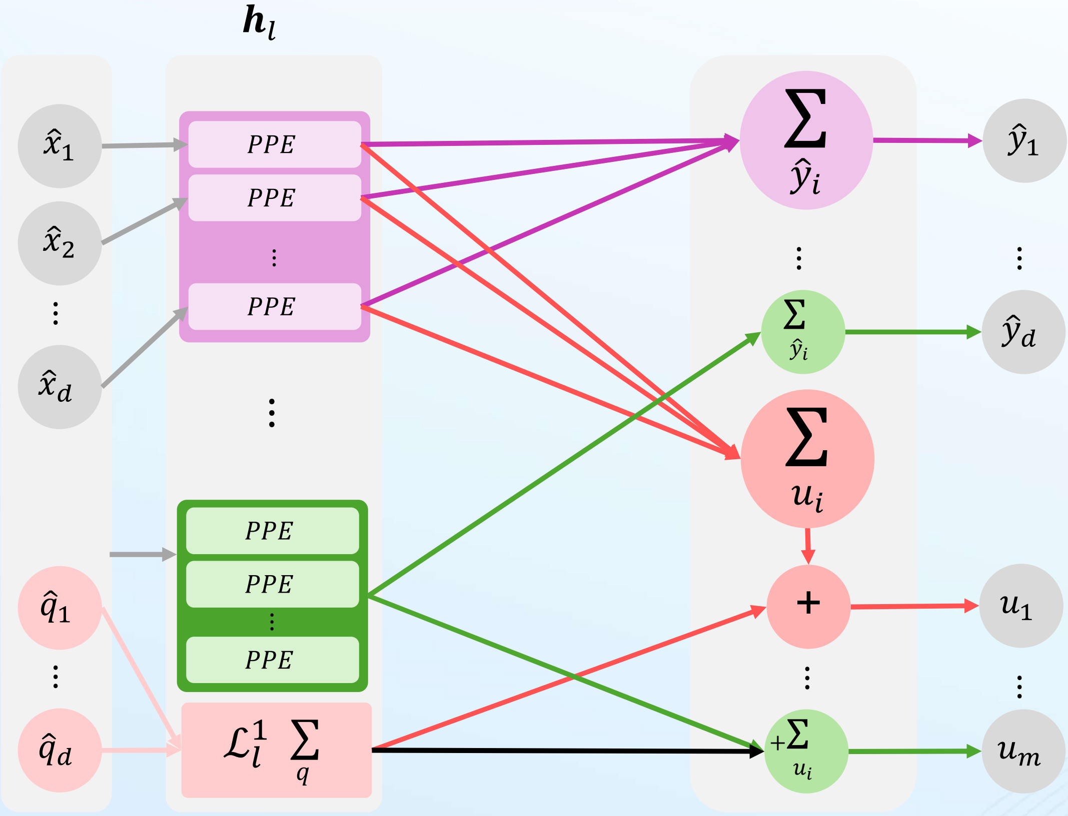



# DAREK




PPE ≡ Piece-wise polynomial error

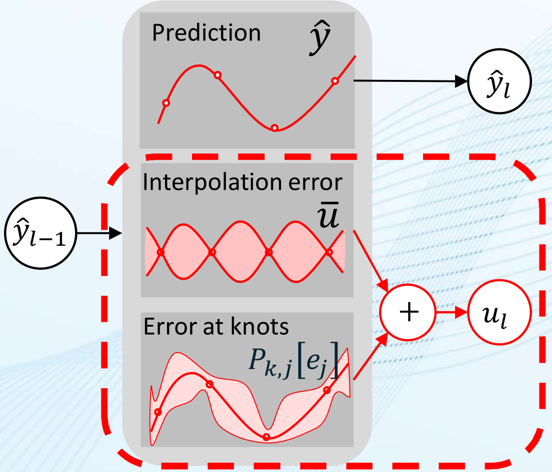
# DAREK




$$|h_l(x) - \hat{h}_l(x)| \leq u_{[l]}(\mathbf{1}_n^\top \hat{\boldsymbol{x}}) + \mathcal{L}_l^1 \mathbf{1}_d^\top \boldsymbol{q}$$

[Thm. 2]

Piecewise Polynomial error $(PPE)$



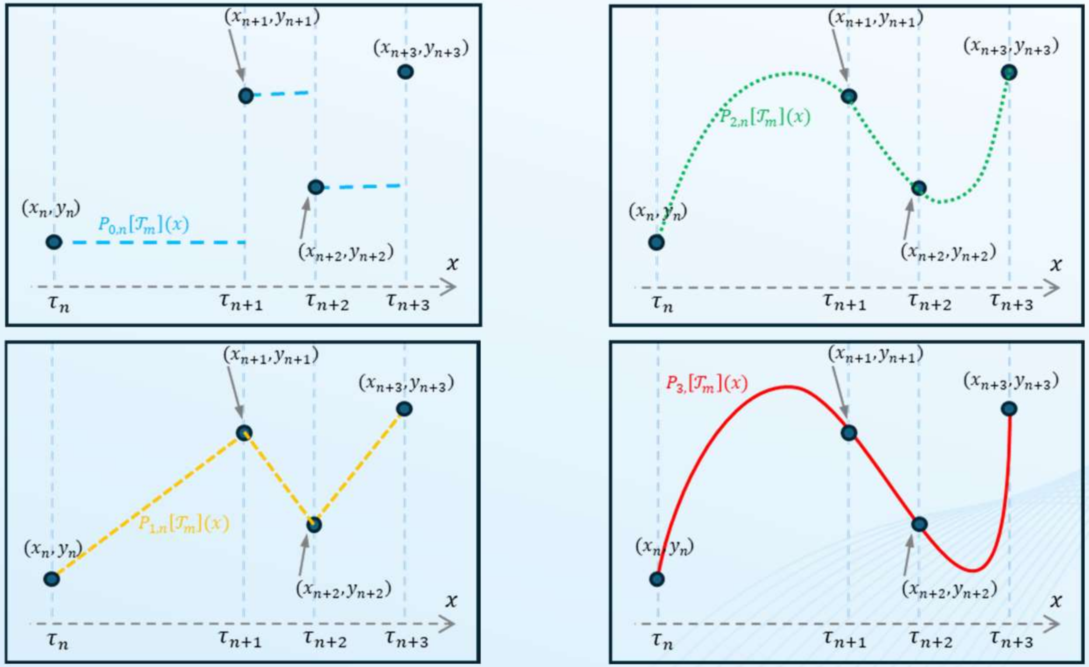

PPE ≡ Piece-wise polynomial error

# Piecewise Newton's Polynomial

$$f(x) \approx \mathcal{P}_{k,n}[f(\tau_{1:m})](x) \coloneqq [\tau_n]f + \sum_{i=1}^{k} [\tau_{n:n+i}]f \prod_{j=n}^{n+i-1} (x - \tau_j)$$

[C. De Boor 1978]

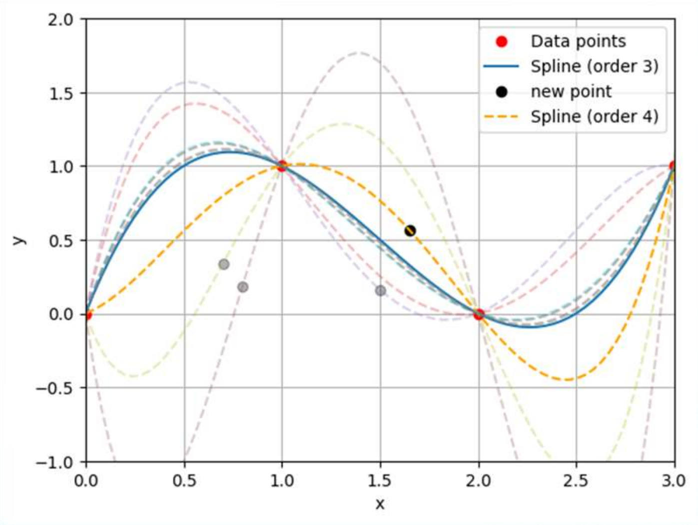
$[.]f \equiv$ Divided differences



# Newton's Polynomial + Reminder

$$f(x) \approx \mathcal{P}_{k,n}[f(\tau_{1:m})](x) \coloneqq [\tau_n]f + \sum_{i=1}^{k} [\tau_{n:n+i}]f \prod_{j=n}^{n+i-1} (x - \tau_j)$$ [C. De Boor 1978]

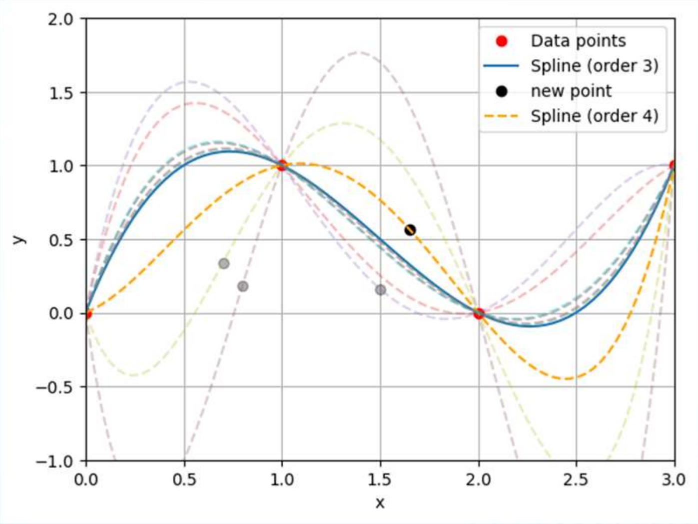
$$f(x) = \mathcal{P}_{k,n}[f(\tau_{1:m})](x) + (x-\tau_n) \dots (x - \tau_{n+k}) [\tau_n, \dots, \tau_{n+k}, x]f$$ [C. De Boor 1978]



# Interpolation error $\bar{u}$

$$f(x) = \mathcal{P}_{k,n}[f(\tau_{1:m})](x) + (x - \tau_n) \dots (x - \tau_{n+k}) [\tau_n, \dots, \tau_{n+k}, x] f$$ [C. De Boor 1978]

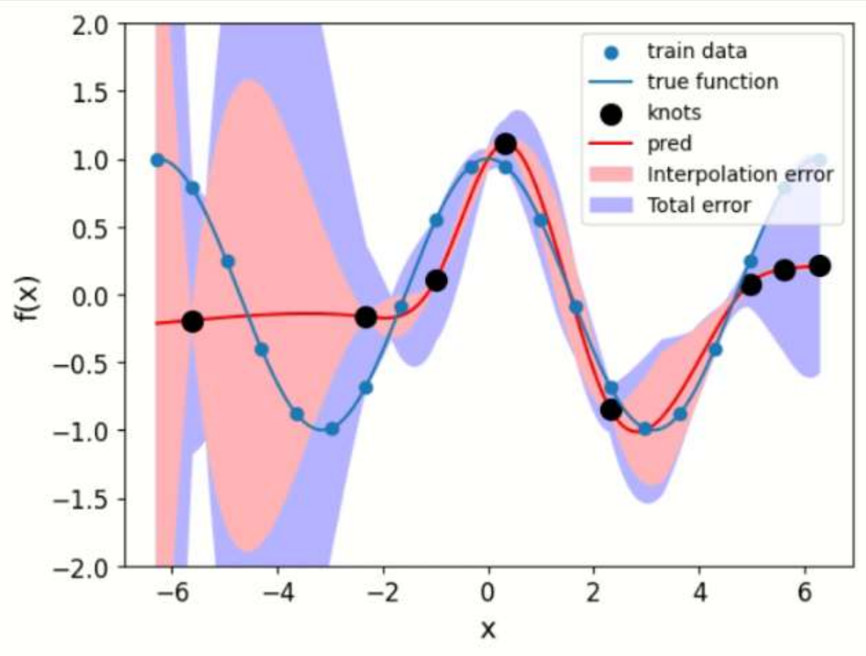
$$\left| f(x) - \mathcal{P}_{k,n}[f(\tau_{1:m})](x) \right| \le \frac{\mathcal{L}_f^{k+1}}{(k+1)!} \left| \prod_{j=n}^{n+i-1} (x - \tau_j) \right| =: \bar{u}_f(x; \tau_{1:m})$$ [Thm. 1]



# Non-zero error at knot

$$e_n^f(\tau_i) \coloneqq f(\tau_i) - \hat{f}_{[n]}(\tau_i)$$

$$\left|f(x) - \hat{f}_{[n]}(x)\right| \le \bar{u}_f(x) + \left|\mathcal{P}_{k,n}\left[e_n^f(\tau_{1:m})\right](x)\right| =: u_f(x;\tau_{1:m})$$ [Lemma 1]



# Lipschitz sharing and error division at knots

- Lipschitz division: $\mathcal{L}_f \Rightarrow \mathcal{L}_h$

$$\mathcal{L}_f = \prod_{l=1}^{L} N_l \, \mathcal{L}_h = (\mathcal{L}_h)^L \prod_{l=1}^{L} N_l \qquad \mathcal{L}_h = \sqrt[L]{\frac{\mathcal{L}_h}{\prod_{l=1}^{L} N_l}}$$

[Inspired by J. Liu 2020]

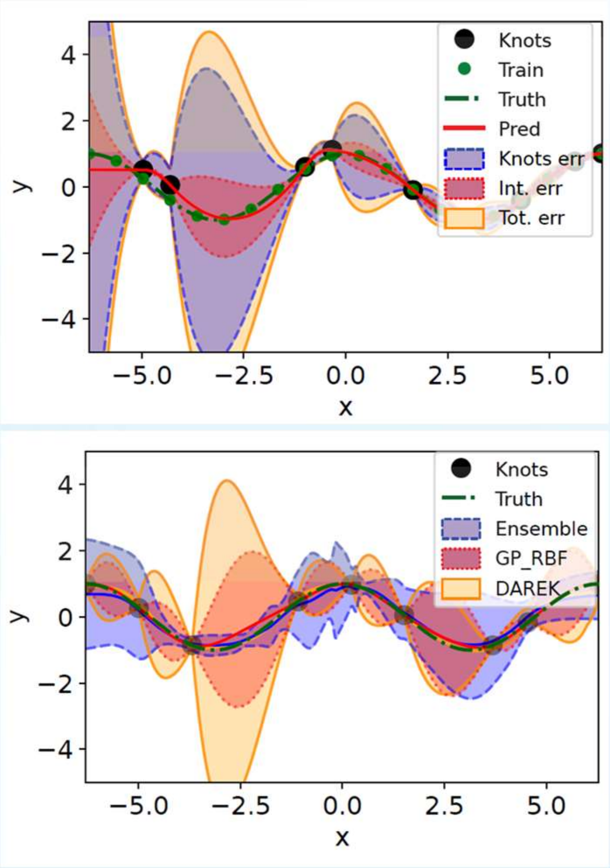
- Error at knot sharing: $e^f \Rightarrow e^h$

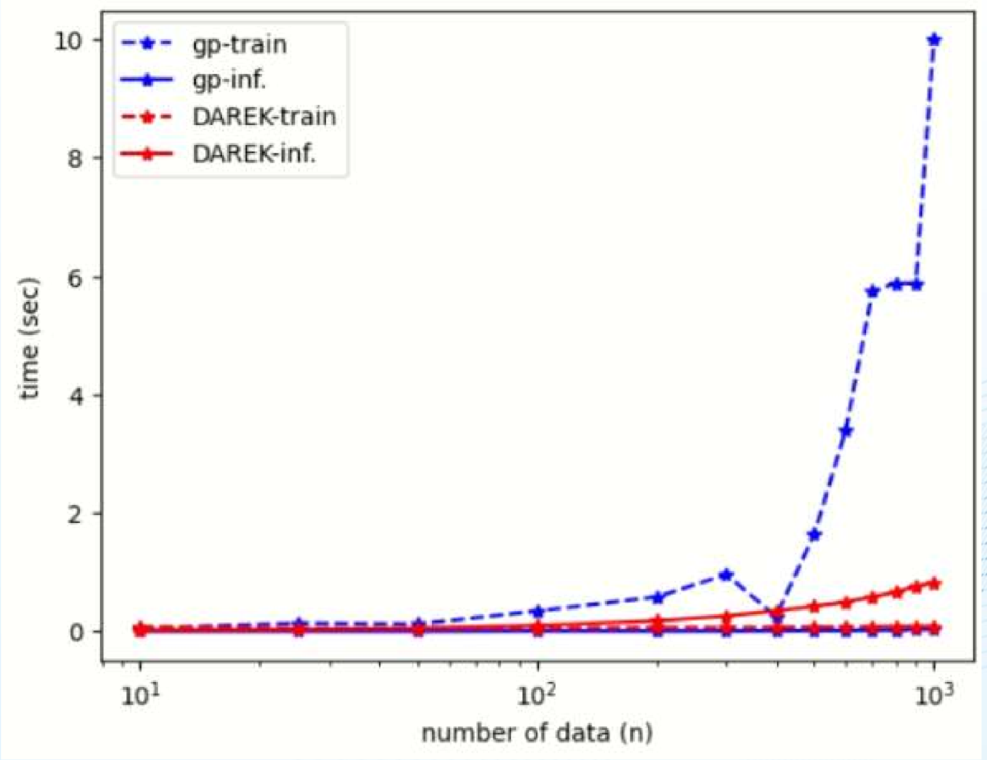
$$e^h(\mathbf{1}^\top \hat{\boldsymbol{g}}(\tau_i)) = e^{g_1}(\tau_i) = \cdots = e^{g_n}(\tau_i) \geq \frac{e^f(\tau_i)}{1 + n\mathcal{L}_h^1}$$

[Assumption 2]

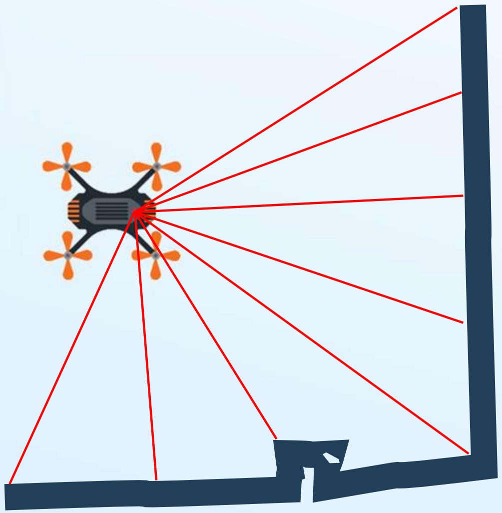
$$e^f(\tau_i) = f(\tau_i) - \hat{f}(\tau_i)$$

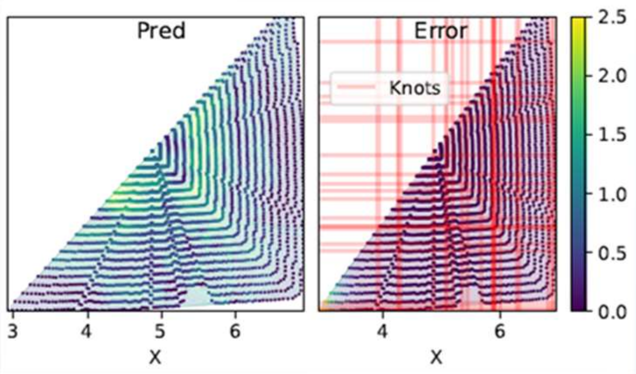
# Experiments

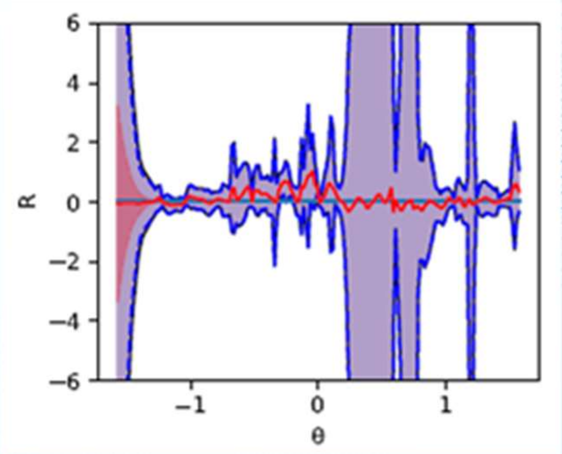


# Experiments





# Conclusion

- **DAREK**, a novel framework for **error estimation** in spline based networks
- Provides **structured, interpretable, and computationally efficient worst-case error bounds**
- Uses **piecewise polynomial error estimation**, ensuring **tight, distance-aware error bounds**

# Future Work

- **Refine Lipschitz division and error propagation** to improve bound tightness
- **Extend DAREK to higher-dimensional problems** for complex applications
- **Integrate DAREK into real-time safe control systems** for autonomous decision-making

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# Thank You